AP Calculus BC / 12

Limits

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	1 – Limits	8 Days

Grade Level Summary

The study of Calculus – BC offers an exciting alternative for our most gifted and advanced students in mathematics. This course permits students to engage in a second year of Calculus study while remaining a part of our student body. This course begins with an overview of limits, as they are the foundational theory for Calculus. The College Board program requires all topics comprising a Calculus AB course to be elements in a BC course as well. This requirement permits us to regain understanding in the three main themes of calculus – Limits, Differentiation and Integration. A Calculus BC course deepens understanding in each area. For example: L'Hopital's Rule is studied to further understand limits; Logistic Differential Equations is one added topic to the area of Differentiation; and, Improper Integrals serves as a sample of how Integration knowledge is expanded. Along with these three main foci, a comprehensive study of Polynomial Approximations and Series is undertaken. But, simply reducing the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations. This course is infused with opportunities to interact with concepts through numerical, graphical and analytical methods. Such an approach strengthens conceptual understanding while supporting the legitimacy of analytical methods. The graphing calculator provides a tool to quickly interact with the study of calculus in all three learning styles. In these investigations, numeric and analytical methods are supported through visual means. This technology provides visual investigations deepening understanding of limits, differential and integral applications.

Grade Level Units

Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title

Limits

Unit Overview

With the aid of technology, graphs of functions are often easy to produce. The emphasis will be on interplay between the numeric, geometric and/or analytic techniques to predict and explain observed local and/or global behavior of a function.

Unit Essential Questions

1. How do we determine the limit of a function?
2. How is asymptotic and unbounded behavior described?
3. How is continuity of a function determined?

Key Understandings

1. Understanding intuitively the limiting process.
2. Estimating limits using graphical resources.
3. Estimating limits using numerical resources.
4. Calculating limits using analytical methods.
5. Investigating asymptotes in terms of graphical behavior.
6. Describing asymptotic behavior in terms of limits involving infinity.
7. An intuitive understanding of continuity of a function.
8. Understanding continuity in terms of limits.

Focus Standards Addressed in the Unit	
CC2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.6	Extend the knowledge of rational functions to rewrite in equivalent forms.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Misconceptions

1. Limits are always a functional value.
2. Limits do not exist if a function does not exist.
3. Limits imply continuity.

Proper Conceptions

1. Limits are what graphs approach … not always what they equal.
2. Since limits are local properties, the *y*-values may be approaching a value where a hole resides.
3. Continuity implies that limits exist; the converse, however, may be false.

Concepts

- Limits Graphically
- Limits Numerically
- Limits Analytically
- Continuity

Competencies

- Determining Limits Numerically.
- Determining Limits Graphically.
- Determining Limits with Algebraic Methods.
- Defining continuity for a function.
- Intermediate and Extreme Value Theorems as a geometric understanding of graphs of continuous functions.
- Evaluating One-Sided limits.
- Analyzing Infinite Limits.
- Use a graphing calculator to explore limits numerically and graphically.

Vocabulary

- Calculus
- Limit
- Tangent Line
- epsilon-delta limit definition
- Continuity
- One-Sided Limit
- Infinite Limits

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction
Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques (Lady Bugs and Limits – an effective tool for presenting limits.)
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:
Medicine – Limit Lab on medicine dosage
Physics Application
Chemistry Application

Additional Resources:
Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:
William C. Witt II

AP Calculus BC / 12

Differentiation

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	2 – Differentiation	14 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title
Differentiation

Unit Overview
Differentiation marks the first fundamental purpose in a dynamic study of mathematics. Connecting the limit concept of pre-calculus mathematics to the tangent line problem permits one to investigate instantaneous rates of change. The unit addresses standardized differentiation techniques such as the power rule, the chain rule and others. These techniques are then connected to the concept of implicit differentiation which leads into additional applications such as related rates.

Unit Essential Questions
1. How does the limit process address the tangent line problem?
2. How do differentiation rules ease the differentiation process?
3. How are these rules applied to higher-order derivatives?
4. How does differentiation offer application through rates of change?
5. What is the role of implicit differentiation in solving related rates problems?

Key Understandings
1. Limit definition of a derivative – tangent line problem.
2. Power Rule for differentiation.
3. Product Rule.
4. Quotient Rule.
5. Chain Rule.
6. Implicit Differentiation.
7. Derivatives as Instantaneous Rates of Change.
8. Derivatives as Slopes of Tangent Lines.
9. Relationship between Differentiability and Continuity.

Focus Standards Addressed in the Unit

CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Important Standards Addressed in the Unit

CC2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

Misconceptions	**Proper Conceptions**
1. Derivatives are separate from limits. 2. Continuity implies differentiability. 3. Average velocity and instantaneous velocity are one in the same.	1. The definition of a derivative is in limit form. 2. Some continuous functions, such as a cusp, have points where a derivative does not exist. 3. Average velocity is slope of a secant line (Pre-calculus) while instantaneous velocity is slope of the tangent line (differential calculus).

Concepts	**Competencies**	**Vocabulary**
• Derivative at a point • Derivative as a function • Second derivative • Instantaneous Rates of Change • Implicit differentiation	• Slope of a Tangent at a point – identification of vertical tangents and points where a derivative does not exist. • Tangent lines to a curve at a point – a local linear approximation. • Approximate a rate of change from graphs and tables of values. • Interpret the derivative as a rate of change in varied applied contexts including velocity, speed and acceleration. • Generate equations involving derivatives. • Derivative presented graphically, numerically, analytically. • Derivative interpreted as an instantaneous rate of change. • Derivative as a slope. • Corresponding characteristics of graphs of a function and the first derivative. • Corresponding characteristics of graphs of a function and the second derivative. • Use graphing calculator to explore the slopes of secant lines and compare with the slope of a tangent line as $\Delta x \to 0$.	• Secant line • Difference Quotient • Tangent line • Slope • Instantaneous rate of change • Derivative notation • Chain Rule • Implicit Differentiation

Assessments

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Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
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Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

Medicine – Limit Lab on medicine dosage
Physics Application
Chemistry Application

Additional Resources:

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Created By:

William C. Witt II

AP Calculus BC / 12

Applications of Differentiation

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	3 – Applications of Differentiation	15 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title
Applications of Differentiation

Unit Overview
Derivative applications enhance a dynamic study of mathematics through calculus. Providing real world situations which rely upon tools for determining derivatives excites learners. These investigations support the need of mechanics while adding validity to their place in the learning process. This unit continues to discuss theory as well. Both Rolle's and the Mean Value theorems are studied.

Unit Essential Questions
1. How is calculus used to determine extrema?
2. How is the first derivative used in describing a function's behavior?
3. How is the second derivative used in describing a function's behavior?
4. How are infinite limits determined?
5. How are first and second derivative concepts applied to real life problems?

Key Understandings
1. Extrema and Critical Numbers.
2. Rolle's Theorem.
3. Mean Value Theorem.
4. Instantaneous Rate of Change as a Limit of Average Rate of Change.
5. First derivative sign connected to increasing/decreasing and extrema points.
6. Second derivative sign connected to concavity/points of inflection.
7. Limits at infinity.
8. Applying derivative data for purpose of curve sketching.
9. Differentiation and optimization problems.
10. Newton's method for approximating zeros.
11. Differentials and their application.

Focus Standards Addressed in the Unit

CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.1.HS.F.5	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
CC.2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.

Important Standards Addressed in the Unit	
CC.2.1.HS.F.1	Apply and extend the properties of exponents to solve problems with rational exponents.
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.
CC.2.2.HS.C.8	Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.

Misconceptions

1. If a derivative does not exist, its graph will not exist.
2. Newton's method is full proof.

Proper Conceptions

1. As long as there is a reversal of sign in a derivative around a *dne* point and the value exists in the original function, an extrema will exist.
2. Newton's Method may fail if an initial condition is near a vertical asymptote.

Concepts

- Velocity
- Speed
- Acceleration
- Increasing
- Decreasing
- Relative extrema
- Concavity
- Points of Inflection

Competencies

- Analysis of curves for monotonic behavior.
- Discuss both Absolute (global) and Relative (local) extrema.
- First derivative sign analysis for the purpose of analyzing characteristics and applications for the original function.
- Second derivative sign analysis for the purpose of analyzing characteristics and applications for the original function.
- Graphically interacting between f, f', and f'' – using both graphing utilities and traditional paper/pencil methods.
- Equations involving the derivatives.
- Interpret derivatives for applications to speed, velocity and acceleration.
- Apply derivative tests to prove optimization in applications.
- Model rates of change, including related rates applications.

Vocabulary

- Relative Extrema
- Critical Numbers
- Rolle's Theorem
- Mean Value Theorem
- Concavity
- Point of Inflection
- Asymptote
- Optimization
- Newton's Method
- Differential

Assessments

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Differentiation:

- Provide graphic organizers
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- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Petroleum
- Auto Industry
- Farming
- Error Estimation for Manufacturing
- Projectile Motion

Additional Resources:

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Created By:

William C. Witt II

AP Calculus BC / 12

Integration

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	4 – Integration	16 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title
Integration

Unit Overview
Integration is the second fundamental topic addressed in calculus. Integration is technically antidifferentiation. So, as the study of calculus continues, students learn how to apply the inverse operation for differentiation – integration. This investigation begins in a general sense – indefinite integration. It then progresses to finding specific numeric solutions with given conditions – definite integration. Definite integrals are determined through application of the Fundamental Theorem of Calculus. This unit also begins to provide study for fundamental applications of the integral.

Unit Essential Questions
1. What is Antidifferentiation?
2. What is Indefinite Integration?
3. How do Riemann Sums result in Definite Integration?
4. What is the Fundamental Theorem of Calculus?
5. How is the Chain Rule of Differentiation applied to Integration?
6. If an antiderivative is inconvenient or impossible to determine, what are some Numerical Integration options?

Key Understandings
1. Antiderivatives as a Family of Functions.
2. Antiderivatives with initial conditions.
3. Antiderivatives and indefinite integration – graphically, numerically and analytical methods.
4. Indefinite Integration with power rule.
5. Areas via Riemann Summation.
6. Definite Integration.
7. Fundamental Theorem of Calculus.
8. Average Value of a Function.
9. Second Fundamental Theorem of Calculus.
10. Integration by Substitution.
11. Numerical Integration by Trapezoidal Rule.
12. Numerical Integration by Simpson's Rule.

Focus Standards Addressed in the Unit

CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.
CC.2.2.HS.D.5	Use polynomial identities to solve problems.
CC.2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.

CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.

Important Standards Addressed in the Unit	
CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

Misconceptions

1. Area is always positive.
2. While integrating with substitution, limits are not important.
3. All integrands have an antiderivative.

Proper Conceptions

1. Pure area is positive … but, area under a rate curve may be negative and represents a drop or loss.
2. It is very important to adjust limits of integration when applying the chain rule to integrands.
3. Many integrands have very difficult or nonexistent solutions. In these cases, numerical integration (Trapezoidal Rule or Simpson's Rule) is a viable solution.

Concepts

- Area under curves
- Riemann Summations
- Limit serves as the foundation for the Integral Process
- Indefinite Integration
- Antiderivatives
- Area under curves
- Definite Integration
- Numerical Integration

Competencies

- Find antiderivatives using basic integration rules.
- Find solutions to indefinite integration when provided initial conditions.
- Calculate definite integrals using various Riemann summation types – Right, Left, Midpoint, Upper, Lower.
- Make connections between Riemann summations and definite integration through the limit process.
- Demonstrate ability to use the Fundamental Theorem of Calculus.
- Demonstrate understanding of basic properties for definite integration.
- Find average value of a function.
- Show use of the second Fundamental Theorem of Calculus
- Demonstrate understanding of particle motion.
- Complete integration that calls for substitution.
- Show ability to solve numerical integration problems using Trapezoidal and Simpson's Rules.
- Calculate the integral form a graph using geometric area formulas, such as a right triangle in the case of a linear absolute value function.
- Use a graphing utility to numerically calculate a definite integral.

Vocabulary

- Antiderivative
- Indefinite Integral
- Summation
- Area of a Plane Region
- Riemann Summation
- Upper Summation
- Lower Summation
- Midpoint Summation
- Definite Integral
- Fundamental Theorem of Calculus
- Trapezoidal Rule
- Simpson's Rule

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

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Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

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Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
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- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Electricity
- Speed of Sound
- Surveying
- Industrial Engineering
- Particle Motion

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus BC / 12

Transcendental Functions and Calculus

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	5 – Transcendental Functions and Calculus	12 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Overview
This unit is strategically positioned. It provides study in transcendental functions answering the void left by the power rule for antidifferentiation. Furthermore, students have been away from the differentiation process for some time. By having the study of transcendental functions fall later in the curriculum, students have an opportunity to reinforce the many derivative procedures studied earlier. The transcendental functions also permit many new areas of application to be addressed.

Unit Essential Questions
1. How do we differentiate the natural logarithmic function?
2. How do integrals apply to the natural logarithmic function?
3. How is the derivative of an inverse function determined?
4. What procedures permit differentiation and integration of exponential functions?
5. What techniques are required for calculus applications to bases other than *e*?
6. How are derivatives and integrals of inverse trigonometric functions calculated?

Key Understandings
1. Expanding differential and integral calculus to the family of logarithmic functions.
2. Expanding differential and integral calculus to the family of exponential functions.
3. Expanding differential and integral calculus to family of inverse trigonometric functions.

Focus Standards Addressed in the Unit	
CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.C.8	Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.

CC.2.2.HS.D.5	Use polynomial identities to solve problems.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.

Misconceptions	**Proper Conceptions**
1. Integrals of the form $\frac{1}{x}$ are not possible because of the power rule failure. 2. Only base *e* logarithms and exponential functions base *e* have calculus application.	1. Transcendental functions offer a solution to integrands of the form $\frac{1}{x}$. 2. Using the *Change of Base Rule*, we can adapt logarithmic and exponential functions into base *e*.

Concepts	**Competencies**	**Vocabulary**
• Differentiation • Integration • Inverse Functions	• Differentiation and Integration of Natural Logarithmic Functions. • Finding inverse functions. • Differentiation and Integration of Natural Exponential Functions. • Differentiation and Integration of Inverse Trigonometric Functions. • Differentiation and Integration of Functions having bases other than *e*. • Use Implicit Differentiation to find the derivative of an inverse function. • Compare relative magnitudes of functions and their rates of change – for example, contrast exponential, polynomial and logarithmic growth models.	• Natural Logarithmic Function • Logarithmic Differentiation • Inverse Function

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

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3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Finance
- Carbon Dating
- Economics
- Chemistry
- Radioactive Half-life
- Sound Intensity
- Heat Transfer

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus BC / 12
Differential Equations

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	6 – Differential Equations	8 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title
Differential Equations

Unit Overview
This unit takes only an elementary investigation into a large array of differential equations. It uses the visual representation of slope fields to help increase understanding of analytical methods. Euler's Method and Separation of Variables are two solution techniques addressed.

Unit Essential Questions
1. What does a slope field represent?
2. How are differential equations solved?
3. What are some elementary applications for first-order differential equations?

Key Understandings
1. Slope fields.
2. Differential Equation solution techniques.
3. Applying differential equations.

Focus Standards Addressed in the Unit	
CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.1.HS.F.4	Use units as a way to understand problems and to guide the solution of multi-step problems.
CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Misconceptions	**Proper Conceptions**
1. Slope Fields are random collections of line segments. 2. Derivatives represented in $\frac{dy}{dx}$ format are inseparable.	1. Each line segment indicates the slope of the tangent line at that point. In other words, replacing *x* and *y* with their values at a point produce a slope value represented as a segment. This graphical representation of a slope field permits one to generalize to a family of functions and/or a specific solution. 2. One can algebraically separate variables of a differential equation for the purpose of finding a general solution.

Concepts	**Competencies**	**Vocabulary**
• Slope Fields • First-order Differential Equations	• Generate a slope field given a differential equation – pencil/paper and on a graphing utility. • Match a slope field to a differential equation. • Match a slope field to its general solution. • Determine a specific solution within a slope field. • Geometric interpretation of differential equations via slope fields and the relationship between slope field and solution curves for differential equations. • Solve differential equations involving Growth and Decay using exponential equations. • Use the separation of variables technique to solve simple first order differential equations. • Determine a numerical solution of a differential equation using Euler's method. • Solve logistic differential equations and use them in modeling applications. • Use a graphing utility to investigate and explain local linearity of slope fiends and determine a solution. • Use a graphing calculator to investigate the family of functions possible in solving indefinite integration.	• Slope Field • Differential Equation

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
3c – Instructional materials and unit project activities engage students in learning.

3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Sailing
- Wildlife Population Growth
- Radioactive Decay
- Forestry
- Intravenous Feeding

Additional Resources:

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AP Calculus BC / 12

Integral Applications

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	7 – Integral Applications	12 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title
Integral Applications

Unit Overview
This unit extends elementary applications of integration into additional key areas. Area of a Plane Region trapped between two curves begins the discussion. Students connect this concept to defining area which is rotated about an axis of revolution thus creating a three dimensional object possessing volume. This unit concludes studying the concepts of arc length and surfaces of revolution. (Upon completion of this unit all AP Calculus AB curricular requirements set forth by the College Board are met.)

Unit Essential Questions
1. What is area between curves?
2. How is the disk method used in determining the volume for a solid of revolution?
3. How is the shell method used in determining the volume for a solid of revolution?
4. How is integration used in determining the length of an arc?
5. How does integration provide us the tools for calculating surface area?

Key Understandings
1. Determine area between curves with respect to either x or y axis.
2. Determine volume of a solid of revolution utilizing both Disk and Shell methods when an area is revolved about an axis.
3. Determine the length of a segment for a function.
4. Determine surface areas generated by curves being revolved about an axis.

Focus Standards Addressed in the Unit

CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Important Standards Addressed in the Unit

CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Misconceptions	**Proper Conceptions**
1. A radius of revolution is determined in only one way. 2. Volume is not connected to area. 3. Lengths of arcs are dependent upon each function. 4. Surface area is a characteristic of regular figures only.	1. Radii of revolutions are determined from the slice to the axis of revolution. 2. Volume is determined through integration of cross-sectional area. 3. Arc length is a summation of chordal lengths processed through the integration process. 4. Surface area utilizes each chordal path being revolved about an axis of revolution summed through the integration process.

Concepts	**Competencies**	**Vocabulary**
• Area of Regions trapped between curves • Volumes of solids of revolution • Arc Length of a function • Surface Area of a revolved irregular figure	• Calculate area trapped between curves utilizing either axis of integration. • Calculate volumes of solids of revolution by the easiest method – Disk or Shell. This will require integration using the appropriate axis of integration dependent upon given criteria. • Calculate arc length using integration techniques. • Calculate surfaces of revolutions. • Calculate volumes of solids with known cross sections. • Solve problems represented verbally.	• Area between curves • Numerical Integration • Axis of integration • Axis of revolution • Disk Method • Washer Method • Shell Method

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction

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3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Building design
- Surveying
- Industrial Engineering
- Electricity
- Environmental Engineering

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
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Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus BC / 12 / Integration Techniques, L'Hopital's Rule, and Improper Integrals

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals	17 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title
Integration Techniques

Unit Overview
This unit begins with a comprehensive review of basic integration techniques designed to reinforce antidifferentiation skills. Integration techniques are then advanced addressing a variety of integration techniques: Integration by Parts, Trigonometric techniques, Partial Fractions. The focus switches to a study of indeterminant limits utilizing L'Hopital's Rule. The unit concludes making connections with L'Hopital's Rule and its application for convergence for Improper Integrals.

Unit Essential Questions
1. How does one solve integrands dealing with basic integration rules?
2. What is integration by parts?
3. How are trigonometric integrals solved?
4. How is trigonometric substitution used in solving some integrands?
5. What is the technique of Partial Fractions?
6. Are there other miscellaneous integration techniques?
7. What is L'Hopital's Rule?
8. What are improper integrals?

Key Understandings
1. Integration using basic rules.
2. Solving integrals that require integration by parts.
3. Solving Trigonometric integrals.
4. Solve integrals using trigonometric substitutions.
5. Use partial fractional decomposition to solve integrands.
6. Solve limits using L'Hopital's Rule.
7. Solve improper integrals.

Focus Standards Addressed in the Unit

CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Misconceptions	Proper Conceptions
1. All integrands have a solution. 2. Solving all integrands represented in terms of *x* is best practice. 3. Limits of undefined expressions are indeterminant. 4. All integrands with a discontinuity are without a solution.	1. Some integrands require numerical methods or are referenced to tables. 2. Some integrands are better solved through a process of trigonometric substitution. 3. L'Hopital's Rule provides us a technique to solve indeterminant forms. 4. Some integrals with an infinite discontinuity have a solution.

Concepts	Competencies	Vocabulary
• Integration Techniques • Application of L'Hopital's Rule	• Develop a wide range of antidifferentiation techniques including substitution of variables and change of limits. • Integration by parts. • Integration of Trigonometric Integrands. • Integration by Trigonometric Substitution. • Integration using Partial Fractional Decomposition. • Solve indeterminate forms of Limits using L'Hopital's Rule. • Solve Improper Integrals that have an infinite limit of integration. • Solve Improper Integrals that have an infinite discontinuity.	• L'Hopital's Rule • Improper Integral

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction

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3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
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3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period

- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Chemistry
- Fluid Force
- Memory Model

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
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Created By:

William C. Witt II

AP Calculus BC / 12

Infinite Series

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	9 – Infinite Series	14 Days

Grade Level Units
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title
Infinite Series

Unit Overview
This unit begins with a discussion of sequences and representation of sequences using subscript notation. The idea a sequence may be defined recursively is also addressed. The main purpose for discussion sequences is to determine whether or not they converge or not. Finite sequences are then extended with an upper limit of infinity. These infinite sequences, termed Series, are then evaluated for convergence or divergence of partial sums. This unit addresses two main categories of series: Series of constants and Taylor series.

Unit Essential Questions
1. What is a series?
2. How does one identify types of series: geometric, harmonic, alternating, *p*-series?
3. Using various tests, how is convergence or divergence of particular series confirmed?
4. How are Taylor and Maclaurin analyzed?
5. How are intervals of convergence and error analysis applied?

Key Understandings
1. Determine the general form of a sequence.
2. Using the general form a sequence, evaluate it for convergence or divergence.
3. Determine the convergence or divergence of a series through evaluation of the limit of the sequence of partial sums including: Telescoping Series, Geometric Series, *p*-Series, Alternating Series.
4. Examine tests for Convergence/Divergence including: N^{th}-Term Test, the Integral Test, Direct Comparison Test, Limit Comparison Test, Ratio Test, Root Test.
5. Taylor Series approximation with graphical illustration of convergence.
6. Use Taylor and Maclaurin polynomials to estimate values of polynomial functions such as e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
7. A Taylor polynomial is centered around a constant $x=c$, while a Maclaurin polynomial is a unique form of a Taylor polynomial centered at $x=0$.
8. Investigate shortcuts in computing Taylor series including: substitution, differentiation, antidifferentiation and the formation of a new series from a known one.
9. Analyze accuracy of approximations with Lagrange error bound.

	10. Use Power Series to represent important functions. 11. Evaluate Power series for radius and interval of convergence.

Focus Standards Addressed in the Unit	
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Misconceptions	**Proper Conceptions**
1. All series are applicable. 2. If a series is convergent, it converges over an interval $(-\infty, \infty)$.	1. In order for a series to be of practical use, it must be convergent – approach a limiting value. 2. Series may be absolute convergent $(-\infty, \infty)$, or it may be convergent over a finite interval and thus applicable only over that given interval.

Concepts	**Competencies**	**Vocabulary**
• Sequence • Series • Convergence • Divergence	• Identify Sequences and their n^{th} term. • Determine convergence or divergence. • Use properties of monotonic and bounded sequences. • Definition of Convergent infinite series. • Use properties of infinite Geometric series. • Use n^{th}-Term Test for Divergence. • Use the Integral Test for convergence/divergence. • Use properties of p-series and harmonic series. • Use the Direct Comparison Test for convergence/divergence. • Use the Limit Comparison Test for convergence/divergence. • Use the Alternating Series Test for convergence. • Use the Alternating Series with error bound. • Classify convergent series as absolutely or conditionally convergent. • Use the Ratio Test for convergence/divergence. • Use the Root Test for convergence/divergence. • Find polynomial approximations of elementary functions and compare them with the elementary function. • Find Taylor and Maclaurin polynomial approximations of elementary functions. • Use the remainder of a Taylor polynomial. • Define a power series. • Find the radius and interval of convergence of a power series. • Differentiate and integrate a power series. • Find a geometric power series that represents a function. • Construct a power series using series operations. • Find a Taylor or Maclaurin series for a function. • Find a binomial series. • Use a basic list of Taylor series to find other Taylor series.	• Sequence • Series • Convergence • Divergence • Recursive • Monotonic Sequence • Bounded Sequence • Infinite Series • Geometric Series • Geometric Series for a Repeating Decimal

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction

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3a – Student assignment sheets communicate expectations for learning.
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Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Investment
- Projectile Motion
- Marketing

Additional Resources:

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AP Calculus BC / 12

Conics, Parametric Equations, and Polar Coordinates

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	10 – Conics, Parametric Equations, and Polar Coordinates	12 Days

Grade Level Units

Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title

Conics, Parametric Equations, and Polar Coordinates

Unit Overview

This unit investigates conic sections and some of their applications through calculus. It begins with an overview of the conic sections and their properties. The next section then investigates many of these plane curves through parametric equations. Such a technique permits the mathematician to represent relations in function format. Once this goal is achieved, the application of calculus techniques is permitted. This unit concludes with a similar investigation of classic functions represented in polar form.

Unit Essential Questions

1. What are the conic sections?
2. What is Eccentricity?
3. What is a parameter?
4. How are functions represented in parametric form?
5. How is calculus applied to parametric equations?
6. What are polar coordinates?
7. How is calculus applied to polar graphs?

Key Understandings

1. Classification of Conic Sections.
2. Anatomy of Conic Sections.
3. Investigate parametric forms of equations.
4. Convert between parametric and rectangular equation forms.
5. Investigate two classic calculus problems: Tautochrome and Brachistochrone problems.
6. Differential applications to parametric equations.
7. Integral applications for parametric equations.
8. Polar form of rectangular equations.
9. Classification of Polar graphs.
10. Differential application to polar equations.
11. Integration of polar form equations.

Focus Standards Addressed in the Unit

CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.
CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.

CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Misconceptions	**Proper Conceptions**
1. Rectangular form of graphing is the best method for representing events. 2. Relations and their naturally occurring events are not applicable under the study of calculus.	1. Parametric and Polar forms of relations offer new methods for graphing and mathematical study. 2. Parametric and Polar forms offer a functional representation of classic relational ordered pairs – permitting the application of calculus techniques.

Concepts	**Competencies**	**Vocabulary**
• Conic Section classification • Parametric form of relations • Calculus of parametric forms • Polar graphing • Calculus of Polar forms	• Classify and sketch conic sections using their properties. • Write equations of conics. • Represent a plane curve though a set of parametric equations. • Convert equations between parametric and rectangular formats. • Find the slope of a tangent line to a curve given by a set of parametric equations. • Find the arc length of a curve given by a set of parametric equations. • Determine the area of a surface of revolution in parametric form. • Demonstrate understanding of the polar coordinate system. • Find the slope of the tangent line to a polar graph. • Find the area of a region bounded by a polar graph. • Find the arc length of a polar graph. • Find the area of surface of revolution in polar form.	• Eccentricity • Parabola • Circle • Ellipse • Hyperbola • Foci • Directrix • Parameter • Polar coordinates

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

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Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction

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Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Antenna Radiation
- Anamorphic Art
- Astronomy
- Architecture

Additional Resources:

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Created By:

William C. Witt II

AP Calculus BC / 12

Vectors and the Geometry of Space

Subject	Grade	Unit	Suggested Timeline
Mathematics	12	11 – Vectors and the Geometry of Space	10 Days

Grade Level Units

Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title

Vectors and the Geometry of Space

Unit Overview

Although not a requirement of the College Board, this unit will lay foundational understanding necessary for the study of calculus applications for Vector-Valued Functions. This unit begins evaluating vectors in a plane. These concepts are extended to three dimensional spaces. The final topic in this unit is to determine cross product of two vectors in space.

Unit Essential Questions

1. What is a vector in a plane?
2. What is a dot product of two vectors?
3. How are vectors represented in space?
4. How does one determine the cross product of two vectors in space?

Key Understandings

1. Vector components and symbols.
2. Perform vector operations.
3. Understand the properties of vector operations.
4. Determine resultant vectors.
5. Use vectors for finding velocity.
6. Determine coordinates in space and distance between them.
7. Force applications through vector analysis.
8. Determine the dot product of two vectors.
9. Find the projection of a vector onto another vector.
10. Find the cross product of two vectors in space.

Focus Standards Addressed in the Unit

CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.
CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Misconceptions	**Proper Conceptions**
1. Vectors are unique to calculus. 2. The study of vectors is "new" and unique.	1. Vectors are used in many applications in physics. 2. Vectors have a foundation in Unit Circle Trigonometry – thus, there is familiar, prior knowledge to build upon.

Concepts	**Competencies**	**Vocabulary**
• Vectors in a plane • Vector operations in a plane • Vectors in space • Operations with vectors in space	• Write the component form of a vector. • Perform vector operations and interpret results. • Write a vector as a combination of standard unit vectors. • Understand the three-dimensional coordinate system. • Analyze vectors in space. • Operations of vectors in space. • Standard Unit Vectors in space. • Use properties of the dot product of two vectors. • Find the direction cosines of a vector in space. • Use vectors to find the work done by a constant force. • Use the triple scalar product of three vectors in space.	• Scalar • Terminal point • Magnitude • Resultant vector • Octants • Unit vector • Dot Product • Cross Product

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (with emphasis on appropriate documentation demonstrating calculus understanding). It is used as a tool for multiple types of assessment. Homework serves as a tool to informally assess need for additional investigation, practice and, at times, as a grade. Students are encouraged to work in pairs and/or groups so that they may orally express concepts, ideas and solutions. A document camera provides a means for students to share and verbally explain their work.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding. As a learning tool, students provide written and verbal explanations of their work to their peers via a document camera.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology. These projects are cooperative by design. Students are required to write justification statements, in complete sentences, that communicate their conceptual understanding of numeric, graphical and analytic procedure. Most projects require students to take screen captures of their graphing calculator results as support of their findings. These screen captures often take multiple forms … numeric (tables), graphical and analytical results.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Navigation
- Architecture
- Work
- Torque

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus BC / 12

Vector-Valued Functions

Subject	**Grade**	**Unit**	**Suggested Timeline**
Mathematics	12	12 – Vector-Valued Functions	10 Days

Grade Level Units

Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques, L'Hopital's Rule, and Improper Integrals
Unit 9 – Infinite Series
Unit 10 – Conics, Parametric Equations, and Polar Coordinates
Unit 11 – Vectors and the Geometry of Space
Unit 12 – Vector-Valued Functions

Unit Title

Vector-Valued Functions

Unit Overview

This unit concludes the required elements of a BC Calculus course as outlined by the College Board. It begins with a study of Vector-Valued functions. The discussion continues with differentiation and integration of Vector-Valued functions. The final discussion addresses the applications of Velocity and Acceleration.

Unit Essential Questions

1. What is a Vector-Valued Function?
2. How are vector-valued functions differentiated?
3. How are vector-valued functions integrated?
4. Using vectors, how is velocity and acceleration calculated?

Key Understandings

1. Sketch a space curve given by a vector-valued function.
2. Extend the concepts of limits and continuity to space curves.
3. Defining the derivative and integral of vector-valued functions.
4. Describe the velocity and acceleration associated with a vector-valued function.

Focus Standards Addressed in the Unit	
CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.
CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Misconceptions	Proper Conceptions
1. Vector-valued functions are "new" applications. 2. The calculus of vector-valued functions is unique.	1. These functions are extensions of plane curves through parametric equations. This concept is extended into the third dimension – another parameter. 2. Derivatives are defined for vector-valued functions with the same difference quotient as standard derivatives – only for each parameter separately.

Concepts	Competencies	Vocabulary
• Vector-Valued Functions • Limits and continuity in space • Differentiation and integration of vector-valued functions	• Analyze a space curve given by a vector-valued function. • Determine the limit of a vector-valued function. • Discuss the continuity of a vector-valued function. • Differentiate a vector-valued function in a plane and space. • Using properties of vector-valued derivatives. • Integrate a vector-valued function in a plane and space. • Use a vector-valued function to analyze projectile motion. • Calculate a position vector by integration.	• Space Curve • Vector-valued Function

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- Physics
- Air Traffic Control

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