

Find all the zeros of the polynomial function.

1. $f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$

2. $f(x) = x^3 + 2x^2 + 4x - 7$

① $\frac{C}{LC} = \pm 1, \pm 3, \pm 9, \pm 27$

② Real zeros: $-3, -1, 3$

$$\begin{array}{r} \textcircled{3} \\ \textcircled{4} \end{array} \left| \begin{array}{ccccc} 1 & 4 & -6 & -36 & -27 \\ \downarrow & -3 & -3 & +27 & 27 \\ -1 & 1 & -9 & -9 & 0 \\ \downarrow & -1 & 0 & 9 \\ 3 & 1 & 0 & -9 & 0 \\ \downarrow & 3 & 9 & & \\ 1 & 3 & 0 & & \end{array} \right.$$

⑤ $x+3=0$

$x = -3$

⑥ All zeros: $x = -3 \text{ (d.r.)}, -1, 3$

① $\frac{C}{LC} = \pm 1, \pm 7$

② Real zeros: 1

$$\begin{array}{r} \textcircled{3} \\ \textcircled{4} \end{array} \left| \begin{array}{cccc} 1 & 1 & 2 & -7 \\ \downarrow & 1 & 3 & 7 \\ 1 & 3 & 7 & 0 \end{array} \right.$$

⑤ $x^2 + 3x + 7 = 0$

Quad.
Form $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-3 \pm \sqrt{9-28}}{2}$
 $a=1$
 $b=3$
 $c=7$ $= \frac{-3 \pm \sqrt{-19}}{2} = \frac{-3 \pm i\sqrt{19}}{2}$

⑥ All zeros: $x = 1, \frac{-3 + i\sqrt{19}}{2}$

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

3. $-7, -4$

$(x+8)(x+4)$ FOIL!

$x^2 + 4x + 8x + 32$

$f(x) = x^2 + 12x + 32$

imaginary zeros come
in pairs

4. $4, i, -i$

$(x-4)(x-i)(x+i)$

$(x-4)(x^2 + i\cancel{x} - i\cancel{x} - i^2)$

$(x-4)(x^2 - (-1))$

$(x-4)(x^2 + 1)$

$x^3 + x - 4x^2 - 4$

$f(x) = x^3 - 4x^2 + x - 4$

5. $-5, 0, -2i, +2i$

$(x+5)(x)(x+2i)(x-2i)$

$(x+5)(x)(x^2 - 2ix + 2(x-4i^2))$

$(x+5)(x)(x^2 - 4(-1))$

$(x^2 + 5x)(x^2 + 4)$

$x^4 + 4x^2 + 5x^3 + 20x$

$f(x) = x^4 + 5x^3 + 4x^2 + 20x$

Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for the function.

6. $h(x) = x^3 - 4x^2 + 5x + 9$

3 total zeros

$\begin{matrix} \uparrow & \uparrow \\ 1 & 2 \end{matrix}$

$$h(-x) = -x^3 - 4x^2 - 5x + 9$$

1 total zero

Pos	Neg	Imag
2	1	0
or		or
0		2

7. $f(x) = x^5 - 6x^4 - 3x^3 + 7x^2 - 8x + 1$

5 total zeros

$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 & 4 \end{matrix}$

$$f(-x) = -x^5 - 6x^4 + 3x^3 + 7x^2 + 8x + 1$$

Pos	Neg	Imag
4	1	0
or		or
2		2
or		or
0		4

Use a graphing calculator to graph the function. Then use the *zero* (or *root*) feature to approximate the real zeros of the function.

8. $g(x) = x^4 + 3x^2 - 2$

$$x = -.75, .75$$

9. $h(x) = x^5 + 12x^3 - 4x^2 + 16x + 25$

$$x = -.86$$

Use a graphing calculator to graph the function. Identify the x -intercepts and points where local maximums or local minimums occur.

10. $f(x) = x^5 - 2x^4 - x^3 + 3x + 1$

$x\text{-int: } -.34, 1.43, 2.1$

local max: $(-.7, 2.44)$

local min: $(1.83, -1.55)$

11. $g(x) = 3x^3 - 6x + x^4$

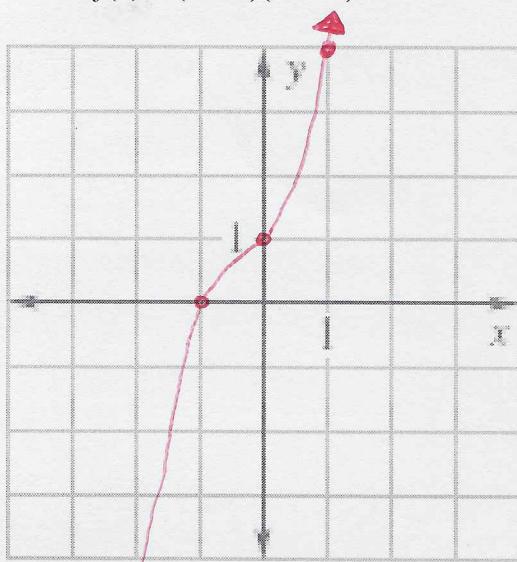
$x\text{-int: } 0, 1.2$

local max: $(-1.19, 4.09)$

local min: $(-1.77, 3.8)$
 $(.71, -2.93)$

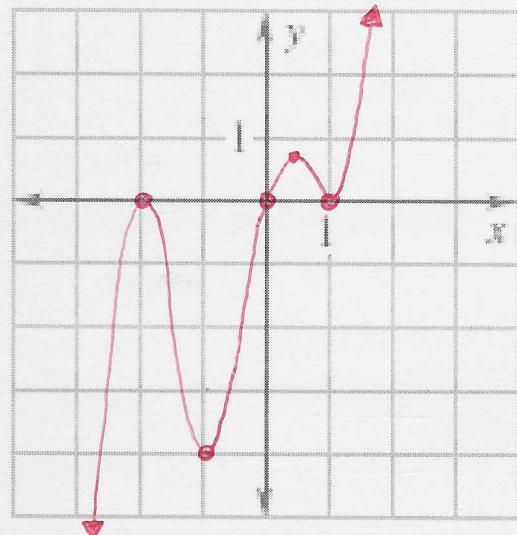
Graph the function.

12. $f(x) = (x + 1)(x^2 + 1)$



X	Y
-2	-5
-1	0
0	1
1	4

13. $g(x) = x(x + 2)^2(x - 1)^2$



X	Y
-2	0
-1	-4
0	0
1	0

local min.: (-1, -4)
local max.: (0, 0)