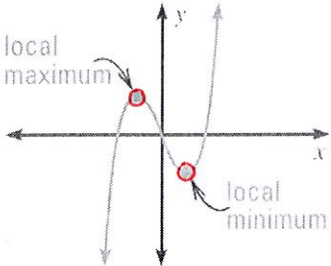


Section:	<b>2 – 8 Analyze Graphs of Polynomial Functions</b>
Essential Question	When does a graph have a local maximum or local minimum?

**Key Vocab:**

<b>Turning Point</b>	<p>Point at which a graph changes from increasing to decreasing or vice versa.</p> <p>→ Think vertex of a parabola.</p> <p><b>Note:</b> Turning points occur on the <math>x</math>-axis when a zero has an even repetition, i.e. double root, quadruple root, etc.</p>	 <p>- Both are turning points</p>
<b>Local Maximum</b>	<p>The <math>y</math>-coordinate of a turning point, if the point is higher than all other nearby points.</p> <p>Typically, it is a turning point on the graph.</p>	
<b>Local Minimum</b>	<p>The <math>y</math>-coordinate of a turning point, if the point is lower than all nearby points.</p> <p>Typically, it is a turning point on the graph.</p>	

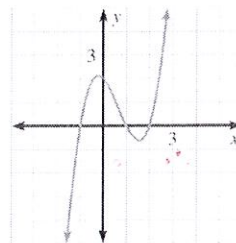
**Key Concept:**

## Turning Points of a Polynomial Function

The graph of every polynomial function of degree  $n$  has **at most**  $n - 1$  turning points.

If a polynomial function has  $n$  distinct real zeroes (no repeated solutions), then its graph has **exactly**  $n - 1$  turning points.

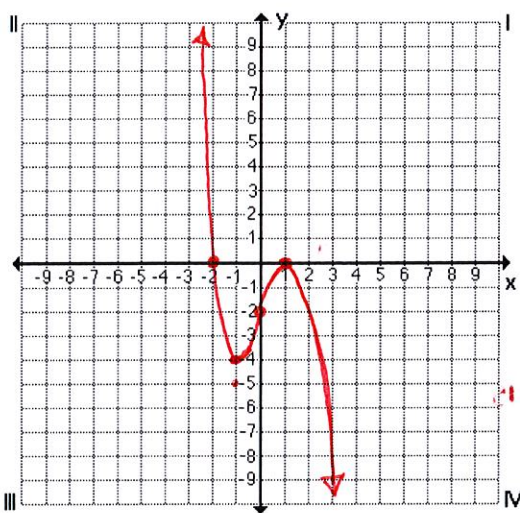
**Example:**  $f(x) = x^3 - 2x^2 - x + 2$



Degree = 3 → 2 Turning Points  
(3 real zeroes)

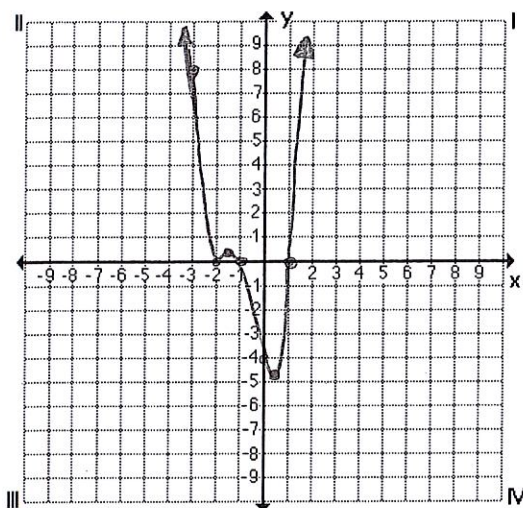
**Show:**

**Ex 1:** Graph the function  $f(x) = -(x+2)(x-1)^2$



x	y
-2	0
-1	-4
0	-2
1	0
2	-4

**Ex 2:** Graph the function  $f(x) = x^4 + 4x^3 + 3x^2 - 4x - 4$ . Identify the  $x$ -intercepts and the points where the local maximums and local minimums occur.



x	y
-3	8
-2	0
-1	0
0	-4
1	0

Local Max:  $(-1.4, 0.3)$

Local Min:  $(-2, 0)$   $(0.4, -4.8)$

$x$ -intercepts:  $-2, -1, 1$

**Ex 3:** You are making a rectangular box out of a 12-inch by 15 inch-piece of cardboard. The box will be formed by making the cuts shown in the diagram and folding up the sides. What is the maximum possible volume for the box?

$$V = l \cdot w \cdot h$$

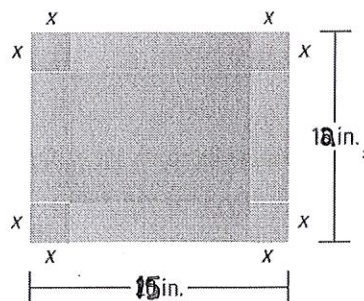
$$V = (15 - 2x)(12 - 2x)x$$

$$V = x(180 - 54x + 4x^2)$$

$$V = 4x^3 - 54x^2 + 180x$$

Maximum occurs at  $x \approx 2.2$

$$\text{Max Volume} \approx 177 \text{ in}^3$$



**Closure:**

- Must the graph of a function always cross the  $x$ -axis at its real zeroes? Explain.

It must hit that point on the  $x$ -axis,  
but not necessarily cross.

