CHAPTER 3 – RATIONAL EXPONENTS AND RATIONAL FUNCTIONS

Big IDEAS:

1) Using rational exponents
2) Performing function operations and finding inverse functions
3) Graphing radical functions and solving radical equations

Section:	**3 – 1 Evaluate *n*th Roots and Use Rational Exponents**
Essential Question	What is the relationship between n^{th} roots and rational exponents?

Key Vocab:

n^{th} Root of a	If $b^n = a$, then b is the n^{th} root of a. **Examples:** $3^2 = 9$ → *3 is the* square *root of* 9 $3 = \sqrt{9}$ $2^4 = 16$ → *2 is the* fourth *root of* 16 $2 = \sqrt[4]{16}$
Rational Exponent	An exponent written in fractional form Represents a radical **Examples:** $4^{1/3} = \sqrt[3]{4^1}$, $5^{2/3} = \sqrt[3]{5^2}$

Key Concept:

Anatomy of a Rational Exponent:	Anatomy of a Radical:
$\underset{\text{base}}{a}^{\overset{\text{exponent}}{n}/\underset{\text{index}}{m}}$	$\sqrt[\overset{\text{index}}{m}]{\underbrace{a}_{\text{radicand}}{}^{\overset{\text{exponent}}{n}}}$

Key Concept:

Real *n*th Roots of *a*		
Let n be an integer $(n > 1)$ and let a be real number.		
$\sqrt[n]{a}$	**The index, n, is an even integer** Example: square root, fourth root, etc.	**The index, n, is an odd integer** Example: cube root, fifth root, etc.
Negative# $a < 0$	No real nth roots (imag.) **Example:** $\sqrt{-1} = i$	One real nth root **Example:** $\sqrt[3]{-8} = -2$
Zero $a = 0$	One real n^{th} root **Example:** $\sqrt{0} = 0$	one real n^{th} root **Example:** $\sqrt[3]{0} = 0$
Positive# $a > 0$	Two real n^{th} roots $(\pm)$ (only when solving) **Example:** $x^2 = 4 \quad x = \pm 2$	one real n^{th} root **Example:** $\sqrt[3]{8} = 2$

Show: EVEN ROOTS ODD ROOTS

Ex 1: Find the indicated real *n*th root(s) of *a*.

a. $n = 5, a = -32$

$\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = -2$

b. $n = 6, a = 1$

$\sqrt[6]{1} = \sqrt[6]{1^6} = 1$

*only include $\pm$ when solving for a variable

Ex 2: Evaluate.

a. $125^{2/3}$

$\sqrt[3]{125^2} = \sqrt[3]{(5^3)^2} = 5^2 = 25$

b. $8^{-4/3}$

$\sqrt[3]{8^{-4}} = \sqrt[3]{(2^3)^{-4}} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

c. $4^{5/2}$

$\sqrt[2]{4^5} = \sqrt[2]{(2^2)^5} = 2^5 = 32$

d. $1^{7/8}$

$\sqrt[8]{1^7} = 1$

Ex 3: Evaluate the expression using a calculator. Round the result to two decimal places.

a. $22^{1/4}$

≈ 2.17

b. $35^{5/6}$

≈ 19.35

c. $\left(\sqrt[5]{11}\right)^4$

≈ 6.81

Ex 4: Solve the equation. Round the result to the nearest hundredth if necessary.

a. $6x^3 = 384$

$\frac{6x^3}{6} = \frac{384}{6}$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$\boxed{x = 4}$

b. $\sqrt[5]{(x-8)^5} = \sqrt[5]{100}$

$x - 8 = 2.51$

$+8 \quad +8$

$\boxed{x = 10.51}$

Ex 5: An exercise ball is made from 7854 square centimeters of material. Find the diameter of the ball. (Use the formula $S = 4\pi r^2$ for the surface area of a sphere.)

$\frac{7854}{4\pi} = \frac{4\pi r^2}{4\pi}$

$\sqrt{625} \approx \sqrt{r^2}$

$r \approx 25$

The diameter is about 50 cm.

Closure:

- Under what circumstances will an equation of the form $(x-k)^n = a$ have **no real solutions**?

  If a is negative, and n is even.

- Under what circumstances will an equation of the form $(x-k)^n = a$ have **exactly one real solution**?

  If n is odd

  or

  If $a = 0$ and n is even

- How do the propertied of square roots related to cube roots, fourth roots, etc?

  Solve by taking the root "opposite" the power

  $x^4 = b \rightarrow$ take the 4^{th} root