Section:	**3 – 2 Apply Properties of Rational FUNctions**
Essential Question	How are the properties of rational exponents related to the properties of integer exponents?

Key Vocab:

Simplest Radical Form	Radicand has no perfect *n*th power factors Denominator is rationalized
Like Radicals	Same **index** AND same **radicand** **Examples:** $5\sqrt[4]{2}$ and $3\sqrt[4]{2}$ $3\sqrt[3]{y}$ and $7\sqrt[3]{y}$ **Note:** You must have *like radicals* in order to add or subtract radicals

Key Concept:

Radicals with Variable Expressions

Remember! When working with real numbers,

EVEN numbered roots only work for POSITIVE numbers.

Since variables can represent positive OR negative numbers, we must consider two cases:

The index, *n*, is an even integer Example: square root, fourth root, etc.	**The index, *n*, is an odd integer** Example: cube root, fifth root, etc.
Example: $\sqrt[4]{3^4} = 3$ ***Non-example*:** $\sqrt[4]{-3^4} = 3i$	**Example:** $\sqrt[7]{5^7} = 5$ **Example:** $\sqrt[7]{(-5)^7} = -5$

Show:

Ex 1: Use the properties of rational exponents to simplify the expression.

a. $12^{1/8} \cdot 12^{5/6}$

$= 12^{(1/8 + 5/6)}$

$= 12^{(3/24 + 20/24)}$

$= \boxed{12^{(23/24)}}$

b. $\left(5^{1/3} \cdot 7^{1/4}\right)^3$

$= 5^{3/3} \cdot 7^{3/4}$

$= 5^1 \cdot 7^{3/4}$

$= \boxed{5 \cdot 7^{3/4}}$

c. $\left(2^6 \cdot 4^6\right)^{-1/6}$

$= 2^{-6/6} \cdot 4^{-6/6}$

$= 2^{-1} \cdot 4^{-1}$

$= \frac{1}{2} \cdot \frac{1}{4}$

$= \boxed{\frac{1}{8}}$

Ex 1 continued: Use the properties of rational exponents to simplify the expression.

d. $\dfrac{10}{10^{2/5}}$

$= 10^{(1 - 2/5)}$

$= 10^{(5/5 - 2/5)}$

$= \boxed{10^{3/5}}$

e. $\left(\dfrac{56^{1/4}}{7^{1/4}}\right)^5$

$= \left(8^{1/4}\right)^5$

$= 8^{5/4}$

$= \left(2^3\right)^{5/4}$

$= \boxed{2^{15/4}}$

f. $\left(\dfrac{20^{1/2}}{5^{1/2}}\right)^3$

$= \left(4^{1/2}\right)^3$

$= 4^{3/2}$

$= \left(2^2\right)^{3/2}$

$= 2^{6/2}$

$= 2^3 = \boxed{8}$

Ex 2: The ratio of the magnitudes of two earthquakes with magnitudes m_1 and m_2 (as measured on the Ritcher scale) is given by the equation $r = \dfrac{10^{m_1}}{10^{m_2}}$. The table gives the magnitudes of some of the largest earthquakes that have occurred in the U.S. How many times stronger was the 1964 quake in Alaska than the 1812 quake in Missouri?

$r = \dfrac{10^{9.2}}{10^{7.9}} = 10^{(9.2-7.9)} = 10^{1.3} \approx 20$

About 20× stronger

Year	State	Magnitude
1812	MO	7.9
1906	CA	7.7
1958	AK	8.3
1964	AK	9.2

Ex 3: Use the properties of radicals to simplify the expression.

a. $\sqrt[3]{125} \cdot \sqrt[3]{8}$

$= 5 \cdot 2$

$= \boxed{10}$

b. $\dfrac{\sqrt[5]{96}}{\sqrt[5]{3}}$

$= \sqrt[5]{\dfrac{96}{3}}$

$= \sqrt[5]{32}$

$= \boxed{2}$

Ex 4: Write the expression in simplest form.

a. $\sqrt[3]{104}$

104 → 52, 2; 52 → 26, 2; 26 → 13, 2

$\boxed{2\sqrt[3]{13}}$

b. $\dfrac{\sqrt[4]{10}}{\sqrt[4]{27}}$

$= \dfrac{\sqrt[4]{10}}{\sqrt[4]{3^3}} \cdot \dfrac{\sqrt[4]{3}}{\sqrt[4]{3}}$

$= \dfrac{\sqrt[4]{30}}{\sqrt[4]{3^4}}$

$= \boxed{\dfrac{\sqrt[4]{30}}{3}}$

Ex 5: Simplify the expression.

a. $7\sqrt[5]{12} - \sqrt[5]{12}$

$= \boxed{6\sqrt[5]{12}}$

b. $4\left(9^{2/3}\right) + 8\left(9^{2/3}\right)$

$= 12(9^{2/3})$

$= 12\sqrt[3]{9^2}$

$= 12\sqrt[3]{(3^2)^2}$

$= 12\sqrt[3]{3^4} = 12\sqrt[3]{3^3 \cdot 3} = 12 \cdot 3\sqrt[3]{3} = \boxed{36\sqrt[3]{3}}$

c. $\sqrt[3]{81} - \sqrt[3]{24}$

$\sqrt[3]{81}$ → 9, 9 → 3 3 3 3 → $3\sqrt[3]{3}$

$\sqrt[3]{24}$ → 6, 4 → 3 2 2 2 → $2\sqrt[3]{3}$

$3\sqrt[3]{3} - 2\sqrt[3]{3} = \boxed{\sqrt[3]{3}}$

d. $\sqrt[3]{5} + \sqrt[3]{40}$

$\sqrt[3]{5}$

$\sqrt[3]{40}$ → 10, 4 → 5 2 2 2 → $2\sqrt[3]{5}$

$\sqrt[3]{5} + 2\sqrt[3]{5} = \boxed{3\sqrt[3]{5}}$

Ex 6: Simplify the expression. Assume all variables are positive.

a. $\sqrt[4]{625z^{12}}$

$\sqrt[4]{625} \cdot \sqrt[4]{z^{12}}$

$5 \quad (z\cdot z\cdot z\cdot z)(z\cdot z\cdot z\cdot z)(z\cdot z\cdot z\cdot z)$

$z \cdot z \cdot z$

$\boxed{5z^3}$

b. $\left(32m^5n^{30}\right)^{1/5}$

$= 32^{1/5} m^{5/5} n^{30/5}$

$= \sqrt[5]{32}\, mn^6$

$= \boxed{2mn^6}$

Ex 7: Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[6]{\dfrac{r^6}{s^{18}}}$

$\dfrac{\sqrt[6]{r^6}}{\sqrt[6]{s^{18}}} = \dfrac{r}{s^{18/6}} = \boxed{\dfrac{r}{s^3}}$

b. $\dfrac{56ab^{3/4}}{7a^{5/6}c^{-3}}$

$= 8a^{(6/6 - 5/6)} b^{3/4} c^3$

$= \boxed{8a^{1/6} b^{3/4} c^3}$

c. $\sqrt[3]{6x^4y^9z^{14}}$

$\sqrt[3]{6}$: $3 \cdot 2$ ← doesn't simplify

$\sqrt[3]{x^4}$: $(x\cdot x\cdot x)\,x$ → $x\sqrt[3]{x}$

$\sqrt[3]{y^9}$: $(y\cdot y\cdot y)(y\cdot y\cdot y)(y\cdot y\cdot y)$ → y^3

$\sqrt[3]{z^{14}}$: $(z\cdot z\cdot z)(z\cdot z\cdot z)(z\cdot z\cdot z)(z\cdot z\cdot z)\, z\cdot z$ → $z^4\sqrt[3]{z^2}$

$\boxed{xy^3z^4\sqrt[3]{6xz^2}}$

d. $\sqrt[7]{\dfrac{p^8}{q^5}}$

$= \dfrac{\sqrt[7]{p^8}}{\sqrt[7]{q^5}}$

$\sqrt[7]{p^8}$: $(p\cdot p\cdot p\cdot p\cdot p\cdot p\cdot p)\,p$ → $p\sqrt[7]{p}$

$= \dfrac{p\sqrt[7]{p}}{\sqrt[7]{q^5}} \cdot \dfrac{\sqrt[7]{q^2}}{\sqrt[7]{q^2}}$

$= \boxed{\dfrac{p\sqrt[7]{pq^2}}{q}}$

Ex 8: Perform the indicated operation. Assume all variables are positive.

(a.) $18\sqrt[3]{u} - 11\sqrt[3]{u}$

*like index & radicand

$\boxed{7\sqrt[3]{u}}$

b. $15a^4b^{2/3} + 8a^4b^{2/3}$

$\boxed{23a^4b^{2/3}}$

(c.) $10\sqrt[4]{5s^7} - s\sqrt[4]{80s^3}$

$\sqrt[4]{s^7}$

$s \cdot s \cdot s \cdot s$ · $s \cdot s \cdot s$

$s\sqrt[4]{s^3}$

$\sqrt[4]{80}$

2, 40, 2, 20, 2, 10, 2, 5

$2\sqrt[4]{5}$

$10s\sqrt[4]{5s^3} - 2s\sqrt[4]{5s^3}$

$\boxed{8s\sqrt[4]{5s^3}}$

d. $\sqrt{9w^5} - w\sqrt{w^3}$

$\sqrt{9}$ → 3

$\sqrt{w^5}$ → $(w \cdot w)(w \cdot w)w$ → $w^2\sqrt{w}$

$\sqrt{w^3}$ → $(w \cdot w)w$ → $w\sqrt{w}$

$3w^2\sqrt{w} - w^2\sqrt{w}$

$\boxed{2w^2\sqrt{w}}$

Closure:

- Explain how to rationalize the denominator for a cube root. How is it different than rationalizing the denominator for a square root? How is it the same?