Section:	**3 – 3 Perform FUNction Operations and Composition**
Essential Question	What operations can be performed on a pair of functions to obtain a third function?

Key Vocab:

Power FUNction	A function of the form $y = ax^b$ where a is a real number and b is a rational number. **Examples:** $y = 4x^{1/2}$, $f(x) = -8\sqrt[3]{x}$
Composition of FUNctions	An operation on functions where $h(x) = (f \circ g)(x) = f(g(x))$. A combination of two functions where one function is performed, then the result is used to perform the second function. The **domain** of h is the set of all x-values such that x is in the domain of f AND $f(x)$ is in the domain of g.

Key Concept:

Operations on FUNctions		
Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.		
Operation	**Definition**	**Example:** $f(x) = 5x, g(x) = x + 2$
Addition	$h(x) = f(x) + g(x)$	$h(x) = 5x + x + 2 = 6x + 2$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 5x - (x+2) = 4x - 2$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = 5x(x+2) = 5x^2 + 10x$
Division	$h(x) = \dfrac{f(x)}{g(x)}$ **Caution:** You cannot divide by zero!	$h(x) = \dfrac{5x}{x+2}$

Domain of the new function h consists of the x-values in the domains of BOTH f and g

Question to Ask: Does x have any restrictions? (Domain)

Show:

Ex 1: Let $f(x) = 5x^{1/3}$ and $g(x) = -11x^{1/3}$. Use the following operations to find $h(x)$ AND then find the domain of $h(x)$

a. $f(x) + g(x) = h(x)$

$h(x) = 5x^{1/3} + -11x^{1/3}$

$h(x) = -6x^{1/3}$

$D: (-\infty, \infty), \mathbb{R}$

	f	g
N	✓	✓
O	✓	✓
P	✓	✓

b. $f(x) - g(x) = h(x)$

$h(x) = 5x^{1/3} - -11x^{1/3}$

$h(x) = 16x^{1/3}$

$D: (-\infty, \infty), \mathbb{R}$

	f	g
N	✓	✓
O	✓	✓
P	✓	✓

Ex 2: Let $f(x) = 8x$ and $g(x) = 2x^{5/6}$. Use the following operations to find $h(x)$, then find the domain of $h(x)$.

a. $f(x) \cdot g(x) = h(x)$

$h(x) = 8x(2x^{5/6})$

$h(x) = 16x^{(6/6 + 5/6)}$

$h(x) = 16x^{11/6}$

$D: x \geq 0, [0, \infty)$

	f	g
N	✓	✗
O	✓	✓
P	✓	✓

b. $\dfrac{f(x)}{g(x)} = h(x)$

$h(x) = \dfrac{8x}{2x^{5/6}}$

$h(x) = 4x^{(6/6 - 5/6)}$

$h(x) = 4x^{1/6}$

$D: x > 0, (0, \infty)$

	f	g
N	✓	✗
O	✓	✓
P	✓	✓

Ex 3: A small company sells computer printers over the internet. The company's total monthly revenue (R) and costs (C) are modeled by the functions $R(x) = 120x$ and $C(x) = 2500 + 75x$, where x is the number of printers sold.

Find $R(x) - C(x)$

$120x - (2500 + 75x)$

$120x - 2500 - 75x = 45x - 2500$

Explain the meaning of this difference.

Revenue − Costs = PROFIT

Ex 4: Let $f(x) = 3x - 4$ and $g(x) = x^2 - 1$. What is the value of $f(g(-3))$? Comp. Functions

$g(-3) = (-3)^2 - 1 = 9 - 1 = 8$ Inside First!

$f(8) = 3(8) - 4 = 24 - 4 = 20$

A. -34   B. 8

C. 20   D. 168

Try: $g(f(-3))$

Domain of Composite Function

1. Find domain of the inside function
2. Find domain of the new function

Ex 5: Let $f(x) = 6x^{-2}$ and $g(x) = 4x + 5$. Find the following compositions AND then find their domains.

← you try

a. $f(g(x))$

$f(4x+5)$

$6(4x+5)^{-2}$

$$\frac{6}{(4x+5)^2}$$

$D: (4x+5)^2 \neq 0$

N	✓
O	✓
P	✓

g

$4x+5 \neq 0$
$\quad -5 \quad -5$

$\dfrac{4x}{4} \neq \dfrac{-5}{4}$　　$\boxed{x \neq \dfrac{-5}{4}}$

b. $g(f(x))$

$g(6x^{-2})$

$4(6x^{-2}) + 5$

$24x^{-2} + 5$

$$\frac{24}{x^2} + 5$$

$f(x) = \dfrac{6}{x^2}$

N	✓
O	✗
P	✓

f

$D: \sqrt{x^2} \neq 0$

$\boxed{x \neq 0}$

c. $g(g(x))$

$g(4x+5)$

$4(4x+5) + 5$

$16x + 20 + 5$

$16x + 25$

$\boxed{D: \mathbb{R}}$

Ex 6: Your starting wage for your part-time job was $6 an hour. All employees get a 5% raise after 6 months. You are given an additional raise of 75-cents per hour as a reward for your outstanding work.

Find your new hourly wage if the 5% raise is applied before the 75-cent raise.

$$6 + 6(0.05) + .75 = \overset{\$}{\underline{7.05}}$$

Find your new hourly wage if the 75-cent raise is applied before the 5% raise.

$$6.75 + 6.75(0.05) = \overset{\$}{\underline{7.09}}$$

Closure:

- When performing a composition of functions, is the order of composition important? If so, which function must be performed first?

- What is the domain of a function and how do you find it?