AP Calculus AB / 11-12 / Preparation for Calculus

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	P – Preliminary for Calculus	7 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Preparation for Calculus

Unit Overview

Students will immediately be called upon to review and refresh mathematical skills from prior courses necessary to bridge prior knowledge with the study of Calculus. Foundations of prior course work are connected to the study of calculus through the limit process. This preparation is designed to highlight techniques and understanding of some key elements from Pre-calculus mathematics utilized within Calculus. A second main purpose of this unit is to familiarize students with the workings of the TI-89 Titanium graphing calculator. This tool is available to every student and is permitted, by the College Board, for use on the AP Calculus AB exam.

Unit Essential Questions

1. How do we describe characteristics of graphs and models?
2. How do we interpret mathematical models of real-life data?
3. How can we represent linear equations?
4. What are functions?
5. How are functions represented?
6. How are functions classified?
7. How are we able to fit mathematical models to data?

Key Understandings

1. Understanding the dynamic elements of calculus is founded in visual representations of an equation.
2. Realizing that solutions, zeros, and roots all carry the same meaning – an x-intercept – is essential.
3. Reviewing the characteristics of Odd and Even functions sets a foundation for understanding some applications in calculus.
4. A skill necessary for calculus applications is determining intersection points of functions.
5. Developing and understanding of mathematical models in graph form is essential.
6. Determine slope given two points.
7. Write equations of lines and graph.
8. Understand the relationship of slopes for parallel and perpendicular lines.
9. Correct use of function notation.
10. Classify functions.
11. Identify various types of transformations of functions.
12. Fitting linear, quadratic and trigonometric models

	appropriately to data.

Focus Standards Addressed in the Unit	
CC2.1.HS.F2	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data.
CC2.2.HS.D.2	Write Expressions in equivalent forms to solve problems.
CC2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

Important Standards Addressed in the Unit	
CC2.2.HS.C.3	Write functions or sequences that model relationships between two quantities.
CC2.2.HS.C.4	Interpret the effects transformations have on functions and find the inverses of functions.
CC2.2.HS.C.5	Construct and compare linear, quadratic, and exponential models to solve problems.

Misconceptions	**Proper Conceptions**
1. Students will struggle recalling the nature of polynomial functions and their graphs. 2. Students will struggle with the meaning of slope in applications. 3. Point/Slope form of a linear equation is extremely helpful in calculus applications. Up to this point, this form of a linear equation has had limited use. They will try and re-memorize this formula as a disjointed piece of information. 4. Analyzing rational functions for zeros and dne points is often a sticking point for students.	1. Review of symmetry properties is helpful … odd and even functions. 2. Having students generate a verbal (written) ratio for the units represented on the horizontal and vertical axes will help them bring meaning to the number which is slope. 3. The point/slope form of a linear equation is easily derived from the slope formula. No need to have an additional memorized formula. 4. Using a single fraction (rational function) , demonstrate the zeros come from the numerator while the dne's come from the denominator.

Concepts	**Competencies**	**Vocabulary**
• Mathematical Models • Analyze Functions • Technology Integration	• Revisit skills and understanding necessary to fit data to an appropriate mathematical model. • Revisit skills and understandings previously presented and still necessary for functions. • Demonstrate the ability to fit a set of data to a mathematical model using a TI-89 graphing calculator.	• Symmetry • Odd Function • Even Function • Rational Functions • Point/Slope Form • Domain • Range • Rational Function • Composite Function • Linear, Quadratic, Trigonometric Regression

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

Statistics – Linear Regression
Physics Application
Chemistry Application
Economic Application
Engineering Application

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus AB / 11-12 / Limits

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	1 – Limits	12 Days

Grade Level Summary

The study of Calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Limits

Unit Overview

With the aid of technology, graphs of functions are often easy to produce. The emphasis will be on interplay between the numeric, geometric and/or analytic techniques to predict and explain observed local and/or global behavior of a function.

Unit Essential Questions

1. How do we determine the limit of a function?
2. How is asymptotic and unbounded behavior described?
3. How is continuity of a function determined?

Key Understandings

1. Understanding intuitively the limiting process.
2. Calculating limits using analytical methods.
3. Estimating limits using graphical and numerical resources.
4. Understanding asymptotes in terms of graphical behavior.
5. Describing asymptotic behavior in terms of limits involving infinity.
6. An intuitive understanding of continuity of a function.
7. Understanding continuity in terms of limits.

Focus Standards Addressed in the Unit

CC2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

Important Standards Addressed in the Unit

CC.2.2.HS.D.6	Extend the knowledge of rational functions to rewrite in equivalent forms.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Misconceptions	**Proper Conceptions**
1. Limits are always a functional value. 2. Limits do not exist if a function does not exist. 3. Limits imply continuity.	1. Limits are what graphs approach … not always what they equal. 2. Since limits are local properties, the *y*-values may be approaching a value where a hole resides. 3. Continuity implies that limits exist; the converse, however, may be false.

Concepts	**Competencies**	**Vocabulary**
• Limits Graphically • Limits Numerically • Limits Analytically • Continuity	• Determining Limits Numerically. • Determining Limits Graphically. • Determining Limits with Algebraic Methods. • Defining continuity for a function. • Intermediate and Extreme Value Theorems as a geometric understanding of graphs of continuous functions • Evaluating one-sided limits. • Analyzing Infinite Limits. • Compare relative magnitudes of functions and their rates of change – for example, contrast exponential, polynomial and logarithmic growth models	• Calculus • Limit • Tangent Line • epsilon-delta limit definition • Continuity • One-Sided Limit • Infinite Limits

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques (Lady Bugs and Limits – an effective tool for presenting limits.)
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:
Medicine – Limit Lab on medicine dosage
Physics Application
Chemistry Application

Additional Resources:
Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:
William C. Witt II

AP Calculus AB / 11-12 / Differentiation

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	2 – Differentiation	20 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Differentiation

Unit Overview

Differentiation marks the first fundamental purpose in a dynamic study of mathematics. Connecting the limit concept of pre-calculus mathematics to the tangent line problems permits one to investigate instantaneous rates of change. The unit addresses standardized differentiation techniques such as the power rule, the chain rule and others. These techniques are then connected to the concept of implicit differentiation which leads into additional applications such as related rates.

Unit Essential Questions

1. How does the limit process address the tangent line problem?
2. How do differentiation rules ease the differentiation process?
3. How are these rules applied to higher-order derivatives?
4. How does differentiation offer application through rates of change?
5. What is the role of implicit differentiation in solving related rates problems?

Key Understandings

1. Limit definition of a derivative – tangent line problem
2. Power Rule for differentiation
3. Product Rule
4. Quotient Rule
5. Chain Rule
6. Implicit Differentiation
7. Derivatives as Instantaneous Rates of Change
8. Derivatives as slopes of tangent lines
9. Relationship between Differentiability and Continuity

Focus Standards Addressed in the Unit

CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Important Standards Addressed in the Unit

CC2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

Misconceptions	**Proper Conceptions**
1. Derivatives are separate from limits 2. Continuity implies differentiability 3. Average velocity and instantaneous velocity are one in the same	1. The definition of a derivative is in limit form 2. Some continuous functions, such as a cusp, have points where a derivative does not exist 3. Average velocity is slope of a secant line (Pre-calculus) while instantaneous velocity is slope of the tangent line (differential calculus)

Concepts	**Competencies**	**Vocabulary**
• Derivative at a point • Derivative as a function • Second derivative • Instantaneous Rates of Change • Implicit differentiation	• Slope of a Tangent at a point – identification of vertical tangents and points where a derivative does not exist • Tangent lines to a curve at a point – a local linear approximation • Approximate a rate of change from graphs and tables of values • Interpret the derivative as a rate of change in varied applied contexts including velocity, speed and acceleration • Generate equations involving derivatives • Derivative presented graphically, numerically, analytically • Derivative interpreted as an instantaneous rate of change • Derivative as a slope • Corresponding characteristics of graphs of a function and the first derivative • Corresponding characteristics of graphs of a function and the second derivative	• Secant line • Difference Quotient • Tangent line • Slope • Instantaneous rate of change • Derivative notation • Chain Rule • Implicit Differentiation

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers

- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:
Medicine – Limit Lab on medicine dosage
Physics Application
Chemistry Application

Additional Resources:
Kahn Academy
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Released AP Test Questions
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Created By:
William C. Witt II

AP Calculus AB / 11-12 / Applications of Differentiation

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	3 – Applications of Differentiation	22 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Applications of Differentiation

Unit Overview

Derivative applications enhance a dynamic study of mathematics through calculus. Providing real world situations which rely upon tools for determining derivatives excites learners. These investigations support the need of mechanics while adding validity to their place in the learning process. This unit continues to discuss theory as well. Both Rolle's and the Mean Value theorems are studied.

Unit Essential Questions

1. How is calculus used to determine extrema?
2. How is the first derivative used in describing a function's behavior?
3. How is the second derivative used in describing a function's behavior?
4. How are infinite limits determined?
5. How are first and second derivative concepts applied to real life problems?

Key Understandings

1. Extrema and Critical Numbers
2. Rolle's Theorem
3. Mean Value Theorem
4. Instantaneous Rate of Change as a Limit of Average Rate of Change
5. First derivative sign connected to increasing/decreasing and extrema points
6. Second derivative sign connected to concavity/points of inflection
7. Limits at infinity
8. Applying derivative data for purpose of curve sketching
9. Differentiation and optimization problems
10. Newton's method for approximating zeros
11. Differentials and their application

Focus Standards Addressed in the Unit

CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.1.HS.F.5	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
CC.2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.

Important Standards Addressed in the Unit	
CC.2.1.HS.F.1	Apply and extend the properties of exponents to solve problems with rational exponents.
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.
CC.2.2.HS.C.8	Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.

Misconceptions	**Proper Conceptions**
1. If a derivative does not exist, its graph will not exist. 2. Newton's method is full proof	1. As long as there is a reversal of sign in a derivative around a dne point and the value exists in the original function, an extrema will exist. 2. Newton's Method may fail if an initial condition is near a vertical asymptote

Concepts	**Competencies**	**Vocabulary**
• Velocity • Speed • Acceleration • Increasing • Decreasing • Relative extrema • Concavity • Points of Inflection	• Analysis of curves for monotonic behavior • Discuss both Absolute (global) and Relative (local) extrema • First derivative sign analysis for the purpose of analyzing characteristics and applications for the original function • Second derivative sign analysis for the purpose of analyzing characteristics and applications for the original function • Graphically interacting between f, f', and f'' • Equations involving the derivatives • Interpret derivatives for applications to speed, velocity and acceleration • Apply derivative tests to prove optimization in applications • Model rates of change, including related rates applications	• Relative Extrema • Critical Numbers • Rolle's Theorem • Mean Value Theorem • Concavity • Point of Inflection • Asymptote • Optimization • Newton's Method • Differential

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Petroleum
- Auto Industry
- Farming
- Error Estimation for Manufacturing
- Projectile Motion

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus AB / 11-12 / Integration

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	4 – Integration	24 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Integration

Unit Overview

Integration is the second fundamental topic addressed in calculus. Integration is technically antidifferentiation. So, as the study of calculus continues, students learn how to apply the inverse operation for differentiation – integration. This investigation begins in a general sense – indefinite integration. It then progresses to finding specific numeric solutions with given conditions – definite integration. Definite integrals are determined through application of the Fundamental Theorem of Calculus. This unit also begins to provide study for fundamental applications of the integral.

Unit Essential Questions

1. What is Antidifferentiation?
2. What is Indefinite Integration?
3. How do Riemann Sums result in Definite Integration?
4. What is the Fundamental Theorem of Calculus?
5. How is the Chain Rule of Differentiation applied to Integration?
6. If an antiderivative is inconvenient or impossible to determine, what are some Numerical Integration options?

Key Understandings

1. Antiderivatives as a Family of Functions
2. Antiderivatives with initial conditions
3. Indefinite Integration with power rule
4. Areas via Riemann Summation
5. Definite Integration
6. Fundamental Theorem of Calculus
7. Average Value of a Function
8. Second Fundamental Theorem of Calculus
9. Integration by Substitution
10. Numerical Integration by Trapezoidal Rule
11. Numerical Integration by Simpson's Rule

Focus Standards Addressed in the Unit

CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.
CC.2.2.HS.D.5	Use polynomial identities to solve problems.

CC.2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.

Important Standards Addressed in the Unit	
CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.

Misconceptions

1. Area is always positive
2. While integrating with substitution, limits are not important
3. All integrands have an antiderivative

Proper Conceptions

1. Pure area is positive … but, area under a rate curve may be negative and represents a drop or loss.
2. It is very important to adjust limits of integration when applying the chain rule to integrands
3. Many integrands have very difficult or nonexistent solutions. In these cases, numerical integration (Trapezoidal Rule or Simpson's Rule) is a viable solution.

Concepts

- Antiderivatives
- Area under curves
- Limit serves as the foundation for the Integral Process
- Definite Integration
- Numerical Integration

Competencies

- Find antiderivatives using basic integration rules
- Find solutions to indefinite integration when provided initial conditions
- Calculate definite integrals using various Riemann summation types – Right, Left, Midpoint, Upper, Lower
- Make connections between Riemann summations and definite integration through the limit process
- Demonstrate ability to use the Fundamental Theorem of Calculus
- Demonstrate understanding of basic properties for definite integration
- Find average value of a function
- Show use of the second Fundamental Theorem of Calculus
- Demonstrate understanding of particle motion
- Complete integration that calls for substitution
- Show ability to solve numerical integration problems using Trapezoidal and Simpson's Rules

Vocabulary

- Antiderivative
- Indefinite Integral
- Summation
- Area of a Plane Region
- Riemann Summation
- Upper Summation
- Lower Summation
- Midpoint Summation
- Definite Integral
- Fundamental Theorem of Calculus
- Trapezoidal Rule
- Simpson's Rule

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction
Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Electricity
- Speed of Sound
- Surveying
- Industrial Engineering
- Particle Motion

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus AB / 11-12 / Transcendental Functions and Calculus

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	5 – Transcendental Functions and Calculus	20 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
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Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Overview

This unit is strategically positioned. It provides study in transcendental functions answering the void left by the power rule for antidifferentiation. Furthermore, students have been away from the differentiation process for some time. By having the study of transcendental functions fall later in the curriculum, students have an opportunity to reinforce the many derivative procedures studied earlier. The transcendental functions also permit many new areas of application to be addressed.

Unit Essential Questions

1. How do we differentiate the natural logarithmic function?
2. How do integrals apply to the natural logarithmic function?
3. How is the derivative of an inverse function determined?
4. What procedures permit differentiation and integration of exponential functions?
5. What techniques are required for calculus applications to bases other than *e*?
6. How are derivatives and integrals of inverse trigonometric functions calculated?

Key Understandings

1. Expanding differential and integral calculus to the family of logarithmic functions.
2. Expanding differential and integral calculus to the family of exponential functions.
3. Expanding differential and integral calculus to family of inverse trigonometric functions.

Focus Standards Addressed in the Unit

CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.C.8	Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.
CC.2.2.HS.D.5	Use polynomial identities to solve problems.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.

Misconceptions	Proper Conceptions
1. Integrals of the form $\frac{1}{x}$ are not possible because of the power rule failure. 2. Only base *e* logarithms and exponential functions base *e* have calculus application.	1. Transcendental functions offer a solution to integrands of the form $\frac{1}{x}$. 2. Using the change of base rule, we can adapt logarithmic and exponential functions into base *e*.

Concepts	Competencies	Vocabulary
• Differentiation • Integration • Inverse Functions	• Differentiation and Integration of Natural Logarithmic Functions • Finding inverse functions • Differentiation and Integration of Natural Exponential Functions • Differentiation and Integration of Inverse Trigonometric Functions • Differentiation and Integration of Functions having bases other than *e* • Use Implicit Differentiation to find the derivative of an inverse function	• Natural Logarithmic Functi • Logarithmic Differentiation • Inverse Function

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3b – Using questioning and Discussion Techniques – connections to implicit differentiation
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples

- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Finance
- Carbon Dating
- Economics
- Chemistry
- Radioactive Half-life
- Sound Intensity
- Heat Transfer

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus AB / 11-12 / Differential Equations

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	6 – Differential Equations	6 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Differential Equations

Unit Overview

This unit takes only an elementary investigation into a large array of differential equations. It uses the visual representation of slope fields to help increase understanding of analytical methods. Euler's Method and Separation of Variables are two solution techniques addressed.

Unit Essential Questions	Key Understandings
1. What does a slope field represent? 2. How are differential equations solved? 3. What are some elementary applications for first-order differential equations?	1. Slope fields 2. Differential Equation solution techniques 3. Applying differential equations

Focus Standards Addressed in the Unit

CC.2.1.HS.F.3	Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
CC.2.1.HS.F.4	Use units as a way to understand problems and to guide the solution of multi-step problems.
CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.

Important Standards Addressed in the Unit

CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.

CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Misconceptions

1. Slope Fields are random collections of line segments
2. Derivatives represented in $\frac{dy}{dx}$ format are in separable.

Proper Conceptions

1. Each line segment indicates the slope of the tangent line at that point. In other words, replacing *x* and *y* with their values at a point produce a slope value represented as a segment. This graphical representation of a slope field permits one to generalize to a family of functions and/or a specific solution.
2. One can algebraically separate variables of a differential equation for the purpose of finding a general solution.

Concepts

- Slope Fields
- First-order Differential Equations

Competencies

- Generate a slope field given a differential equation.
- Match a slope field to a differential equation
- Match a slope field to its general solution
- Determine a specific solution to a slope field
- Geometric interpretation of differential equations via slope fields and the relationship between slope field and solution curves for differential equations
- Solve differential equations involving Growth and Decay using exponential equations
- Use the separation of variables technique to solve simple first order differential equations

Vocabulary

- Slope Field
- Differential Equation

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction

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3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Sailing
- Wildlife Population Growth
- Radioactive Decay
- Forestry
- Intravenous Feeding

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
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Created By:

William C. Witt II

AP Calculus AB / 11-12 / Integral Applications

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	7 – Integral Applications	18 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Integral Applications

Unit Overview

This unit extends elementary applications of integration into additional key areas. Area of a Plane Region trapped between two curves begins the discussion. Students connect this concept to defining area which is rotated about an axis of revolution thus creating a three dimensional object possessing volume. This unit concludes studying the concepts of arc length and surfaces of revolution. (Upon completion of this unit all AP Calculus AB curricular requirements set forth by the College Board are met.)

Unit Essential Questions

1. What is area between curves?
2. How is the disk method used in determining the volume for a solid of revolution?
3. How is the shell method used in determining the volume for a solid of revolution?
4. How is integration used in determining the length of an arc?
5. How does integration provide us the tools for calculating surface area?

Key Understandings

1. Determine area between curves with respect to either *x* or *y* axis
2. Determine volume of a solid of revolution utilizing both Disk and Shell methods when an area is revolved about an axis
3. Determine the length of a segment for a function
4. Determine surface areas generated by curves being revolved about an axis

Focus Standards Addressed in the Unit

CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Important Standards Addressed in the Unit	
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Misconceptions	Proper Conceptions
1. A radius of revolution is determined in only one way 2. Volume is not connected to area 3. Lengths of arcs area are function dependent 4. Surface area is a characteristic of regular figures only	1. Radii of revolutions are determined from the slice to the axis of revolution 2. Volume is determined through integration of cross-sectional area … 3. Arc lengths are a summation of chordal lengths processed through the integration process 4. Surface are utilize each chordal path being revolved about an axis of revolution summed through the integration process

Concepts	Competencies	Vocabulary
• Area of Regions trapped between curves • Volumes of solids of revolution • Arc Length of a function • Surface Area of a revolved irregular figure	• Calculate area trapped between curves utilizing either axis of integration. • Calculate volumes of solids of revolution by the easiest method – Disk or Shell. This will require integration using the appropriate axis of integration dependent upon given criteria. • Calculate arc length using integration techniques. • Calculate surfaces of revolutions.	• Area between curves • Numerical Integration • Axis of integration • Axis of revolution • Disk Method • Washer Method • Shell Method

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

Suggested Strategies to Support Design of Coherent Instruction

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Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples

- Permit projects to be completed over extended time period
- Provide lesson notes via visual presentation (smart board) as well as in notebook formats

Interdisciplinary Connections:

- Building design
- Surveying
- Industrial Engineering
- Electricity
- Environmental Engineering

Additional Resources:

Kahn Academy
Textbook Ancillary Materials
College Board AP Course Guidelines
Released AP Test Questions
www.collegeboard.org

Created By:

William C. Witt II

AP Calculus AB / 11-12 / Integration Techniques

Subject	Grade	Unit	Suggested Timeline
Mathematics	11/12	8 – Integration Techniques	20 Days

Grade Level Summary

The study of calculus has its focus around three areas. The first is foundational preparation – developing basics in Algebra, Geometry, Statistics and Trigonometry (Pre-calculus). Bridging the span between the studies of Pre-calculus into Calculus is Limits. By adding the limit process to foundational skills and understanding, two main problems of calculus can be explored – differentiation and integration. It is the limit process that takes the static study of pre-calculus mathematics and permits dynamic application to real-life situations through calculus. It is advantageous for students to approach the study of calculus through these three themes – Limits, Differentiation and Integration. Limiting the study of calculus to the memorization of facts and rules truly limits one's ability to apply mathematics to real-life situations.

Grade Level Units

Unit P – Preparation for Calculus
Unit 1 – Limits
Unit 2 – Differentiation
Unit 3 – Applications of Differentiation
Unit 4 – Integration
Unit 5 – Transcendental Functions and Calculus
Unit 6 – Differential Equations
Unit 7 – Integral Applications
Unit 8 – Integration Techniques

Unit Title

Integration Techniques

Unit Overview

This unit is an extension for AP Calculus Students in the AB course. It is typically presented following the administration of the AP Exams by the College Board. Since the bulk of our AP students are entering collegiate studies that require two semesters of Calculus, we use this time to extend our study beyond the prescribed AB course descriptions in preparation for their future needs. This unit addresses a variety of integration techniques as well as indeterminant limits.

Unit Essential Questions

1. How does one solve integrands dealing with basic integration rules?
2. What is integration by parts?
3. How are trigonometric integrals solved?
4. How is trigonometric substitution used in solving some integrands?
5. What is the technique of Partial Fractions?
6. Are there other miscellaneous integration techniques?
7. What is L'Hopital's Rule?

Key Understandings

1. Integration using basic rules.
2. Solving integrals that require integration by parts.
3. Solve integrals using trig substitutions.
4. Use partial fractional decomposition to solve integrands.
5. Use integral tables when necessary.
6. Solve limits using L'Hopital's Rule

Focus Standards Addressed in the Unit

CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.9	Use reasoning to solve equations and justify the solution method.

Important Standards Addressed in the Unit

Misconceptions	**Proper Conceptions**
1. All integrands have a solution 2. Solving all integrands represented in terms of *x* is best practice?	1. Some integrands require numerical methods or are referenced to tables 2. Some integrands are better solved through a process of trigonometric substitution

Concepts	**Competencies**	**Vocabulary**
• Integration Techniques • Application of L'Hopital's Rule	• Develop a wide range of antidifferentiation techniques • Integration by parts • Integration by Trig Substitution • Integration using Partial Fractional Decomposition • Solve Indeterminate Forms of Limits	• L'Hopital's Rule

Assessments

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Differentiation:

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Interdisciplinary Connections:

- Chemistry
- Fluid Force
- Memory Model

Additional Resources:

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