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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 1** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**2 -3 Weeks**<br>**(Approximately 15 days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | **Unit 1 - Getting Started**<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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| **Unit Title** | Unit 1 – Getting Started |
| **Unit Summary** | Statistics is the study of how to collect, organize, analyze and interpret numerical information from populations or samples. This unit introduces some basic terminology associated with the study of statistics and discusses both the different sampling methods used to collect data and different ways to classify data. The unit addresses the difference between an observational study and an experiment highlighting the importance of completely randomized experiments. Finally concerns of bias and ethical practices in statistics are discussed. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. Why is statistics important?<br>2. What is the nature of data?<br>3. How can you draw a random sample?<br>4. What are other sampling techniques?<br>5. How can you design ways to collect data?<br>6. Why are ethical considerations important? | 1. Use statistical terminology appropriately.<br>2. Identify variables in a statistical study.<br>3. Distinguish between quantitative and qualitative variables, populations and samples, and parameters and statistics.<br>4. Determine the level of measurement of data.<br>5. Explain the difference between descriptive and inferential statistics.<br>6. Describe different types of sampling.<br>7. Explain the importance of simple random sampling. |

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| | 8. Demonstrate how to construct a simple random sample.<br>9. Simulate a random process.<br>10. Explain what a census is.<br>11. Explain the difference between observational studies and experiments.<br>12. Recognize the components of a completely randomized experiment.<br>13. Explain potential pitfalls that might make data from surveys unreliable.<br>14. Recognize requirements for ethical practices in statistics. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.4.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable |
| **CC.24.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.4.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable |
| **CC.24.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● Data for a study can be collected without following a random sampling process. That is, data that is not collected using a random sampling process will still yield reliable results.<br>● Results from sample data are applicable to any population.<br>● Simple random samples are defined as samples where each individual has an equal chance of being selected (Students often perceive this definition as adequate but this is not a correct definition.).<br>● Observational studies and experiments are both useful for studying the effects of a treatment on a variable. | ● It is important to think about the research objectives prior to collecting data and then develop a plan for collecting the data that involves random sampling. Data that is collected without the use of random sampling will yield unreliable results.<br>● When you work with sample data, you must carefully consider the population from which they are drawn. Observations and analysis of the sample are applicable to only the population from which the sample is drawn.<br>● The correct definition of a simple random sample states that <u>every sample</u> of a specified size (n) from the population has an equal chance of being selected (which implies that each individual also (sample of size n = 1) has an equal chance of being selected).<br>● Only experiments can be used to study the effects of a treatment on a variable as observational studies do not impose a treatment on a variable. |

| Knowledge & Concepts | Skills & Competencies | Dispositions & Practices |
|---|---|---|
| • Quantitative vs. Qualitative<br>• Population vs. Sample<br>• Parameter vs. Statistic<br>• Levels of Measurement<br>• Descriptive vs. Inferential Statistics<br>• Sampling Techniques<br>• Procedure for planning a statistical study<br>• Experiments vs. Observation<br>• Characteristics of experimental design<br>• Guidelines for planning a statistical study<br>• Data Collection Plan<br>• Limitations of Surveys<br>• Importance of Ethics | • Use statistical terminology appropriately.<br>• Distinguish between quantitative and qualitative data.<br>• Distinguish between a population and a sample.<br>• Distinguish between a parameter and a statistic.<br>• Identify the level of measurement for a set of data.<br>• Identify, describe, and perform various sampling methods.<br>• Explain the difference between an experiment and an observational study.<br>• Follow basic guidelines for planning a statistical study.<br>• Explain the key characteristics of experimental design.<br>• State the important features of a data collection plan.<br>• Recognize the importance of ethical practice in statistics. | • Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>• Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>• Students will evaluate their own understanding during homework reviews.<br>• Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>• Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>• Students will be assessed informally on a daily basis through student / teacher interactions.<br>• Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>• Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| • Statistics<br>• Individuals<br>• Variable<br>• Categorical Variable<br>• Quantitative Variable<br>• Qualitative Variable<br>• Population Data<br>• Sample Data<br>• Population Parameter<br>• Sample Statistic<br>• Nominal Level of Measurement<br>• Ordinal Level of Measurement<br>• Interval Level of Measurement<br>• Ratio Level of Measurement<br>• Descriptive Statistic | • Simple Random Sample<br>• Random Number Table<br>• Simulation<br>• Sampling with replacement<br>• Stratified Sampling<br>• Systematic Sampling<br>• Cluster Sampling<br>• Multistage Sampling<br>• Convenience Sampling<br>• Sampling Frame<br>• Undercoverage<br>• Sampling Error<br>• Nonsampling Error<br>• Census<br>• Sample | • Experiment<br>• Placebo Effect<br>• Treatment Group<br>• Completely Randomized Experiment<br>• Block<br>• Randomized Block Experiment<br>• Control Group<br>• Randomization<br>• Replication<br>• Double-Blind Experiment<br>• Survey<br>• Nonresponse<br>• Hidden Bias<br>• Voluntary response<br>• Lurking variable |

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| ● Inferential Statistics | ● Observational Study | ● Confounding Variable |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Advertising
- Archaeology
- Business and Marketing
- Ecology
- Education
- Fishing
- Health Care
- Student Life

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**

Thomas A. Seltzer

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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 2** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**2 – 3 Weeks**<br>**(Approximately 15 days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>**Unit 2 - Organizing Data**<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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| **Unit Title** | Unit 2 - Organizing Data |
| **Unit Summary** | This unit focuses on the descriptive branch of statistics that involves the organization and display of data. Various graphs can be used to display data and show how the data are distributed. In this chapter, several different graphical displays are discussed; frequency distributions, histograms, ogives, dotplots, bar graphs, pareto charts, circle graphs, time series graphs, and stem-and-leaf displays. It is important to be able to determine which type of graph is appropriate for a given data set and to be able to correctly construct and interpret each type of graph. The common shapes of histograms and their associated population distributions are also presented and discussed. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. What are histograms? When are they used?<br>2. What are common distribution shapes?<br>3. How can you select graphs appropriate for given data sets?<br>4. How can you quickly order data and, at the same time, reveal the distribution shape? | 1. Use a frequency table to organize data.<br>2. Construct histograms, frequency histograms and ogives.<br>3. Identify and describe basic distribution shapes: uniform, symmetric, skewed and bimodal.<br>4. Determine which type of graph is appropriate for representing specific data sets.<br>5. Construct bar graphs, Pareto charts, circle graphs and time-series graphs.<br>6. Construct a stem-and-leaf display for a given data set. |

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| | 7. Compare stem-and-leaf displays to histograms.<br>8. Use stem-and-leaf displays to visualize data distributions.<br>9. Interpret graphical displays. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.1.HS.F.3** | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.1.HS.F.3** | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● Histograms and bar graphs are essentially the same.<br>● Students often misinterpret the meaning of skewed left and skewed right. Students often think distributions that are skewed left have a mound to the left and distributions that are skewed right have a mound to the right. Students also sometimes confuse the idea of positively skewed and negatively skewed distributions for this reason.<br>● Students may be tempted to construct graphical displays that are artistic and consequently distort the meaning of the data.<br>● Gaps in histograms between bars signify the presence of outliers.<br>● Any type of graph can be used to display data. | ● Histograms and bar graphs are unique and are constructed using different procedures.<br>● Distributions that are skewed left or negatively skewed have the longer "tail" trailing to the left with the majority of the data to the right. Distribution that are skewed right or positively skewed have the longer "tail" trailing to the right with the majority of the data to the left.<br>● While it is okay to use some artistry in constructing graphical displays, it is important that the data not be distorted and the graph be easy to read.<br>● Gaps in histograms between bars only suggest the possibility that outliers may be present. A more in depth process for determining outliers will be discussed in a future unit.<br>● The type of graph used to display data depends on the type of data being displayed. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Frequency tables and relative frequency tables<br>● Histograms and relative frequency histograms<br>● Shapes of distributions<br>● Cumulative frequency tables and ogives<br>● Dotplots<br>● Bar graphs | ● Determine the class width.<br>● Tally data.<br>● Determine class boundaries.<br>● Determine class midpoints.<br>● Construct a frequency table.<br>● Construct a relative frequency table.<br>● Construct and interpret histograms.<br>● Construct and interpret relative frequency histograms. | ● Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>● Students will be required to complete |

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| ● Circle graphs<br>● Time-series graphs<br>● Determining appropriate graphs<br>● Interpreting graphs<br>● Stem-and-leaf displays | ● Define and recognize the common shapes of distributions.<br>● Construct cumulative-frequency tables.<br>● Construct and interpret ogives.<br>● Construct and interpret dotplots.<br>● Construct and interpret bar graphs and Pareto charts.<br>● Construct and interpret circle graphs.<br>● Construct and interpret time-series graphs.<br>● Determine the appropriate graph for representing a set of data.<br>● Construct and interpret stem-and-leaf displays.<br>● Use graphical displays to locate outliers.<br>● Use technology to construct graphical displays. | various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>● Students will evaluate their own understanding during homework reviews.<br>● Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>● Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>● Students will be assessed informally on a daily basis through student / teacher interactions.<br>● Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>● Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

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|---|---|---|
| ● Frequency Table<br>● Frequency Distribution<br>● Class Width<br>● Class Lower Limit<br>● Class Upper Limit<br>● Class Frequency<br>● Class Midpoint<br>● Class Mark<br>● Class Boundaries<br>● Relative Frequency<br>● Relative Frequency Table<br>● Histogram<br>● Relative Frequency Histogram | ● Mound-Shaped Symmetric Distribution<br>● Uniform Distribution<br>● Skewed Left<br>● Skewed Right<br>● Bimodal Distribution<br>● Outliers<br>● Cumulative Frequency<br>● Ogive<br>● Dotplot<br>● Bar Graph<br>● Cluster Bar Graph | ● Changing Scale<br>● Pareto Chart<br>● Circle Graphs or Pie Charts<br>● Time-Series Graphs<br>● Time Series<br>● Donut Pie Chart<br>● EDA<br>● Stem-and-Leaf Display<br>● Stem<br>● Leaf<br>● Split Stem<br>● Back-to-Back Stem Plot |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Advertising
- Agriculture

- Archaeology
- Commercial Fishing
- Criminal Justice
- Driving
- Ecology
- Education
- Health / Health Care
- Law
- Law Enforcement
- Lifestyle
- Medical Science
- Sports
- Technology

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**

Thomas A. Seltzer

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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 3** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**1 – 2 Weeks**<br>**(Approximately 14 days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>**Unit 3 – Averages and Variation**<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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| **Unit Title** | Unit 3 – Averages and Variation |
| **Unit Summary** | This unit continues to focus on the descriptive branch of statistics. Unlike the previous unit that sought to provide methods to organize and display data, this unit provides methods for summarizing data. This unit will introduce methods for calculating the various measures of central tendency; the mean, median, mode, trimmed mean and weighted average. Methods for calculating measures of spread including the variance, standard deviation and range are demonstrated. Chebyshev's Theorem enables us to estimate the data spread about the mean. The coefficient of variation will allow comparisons of the relative spread of different data sets. Box-and-whisker plots will be used to show how data are distributed about the median. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. What are commonly used measures of central tendency? What do they tell you?<br>2. How do variance and standard deviation measure data spread? Why is this important?<br>3. How do you make a box-and-whisker plot, and what does it tell about the spread of the data? | 1. Compute and interpret the mean, median and mode of a given data set.<br>2. Explain how measures of center are affected by extreme outliers.<br>3. Compute trimmed means and weighted averages.<br>4. Compute the range, variance and standard deviation for a given data set.<br>5. Compute and interpret the coefficient of variation for a |

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| | given data set.<br>6. Apply Chebyshev's Theorem.<br>7. Interpret percentile scores.<br>8. Compute the five-number summary for a given data set.<br>9. Construct box-and-whisker plots.<br>10. Describe how a box-and-whisker plot indicates spread of data about the mean. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● Trimmed mean is calculated by simply removing a percentage of data from the overall data set. For example, a 5% trimmed mean would be calculated by removing 5% of the data, 2.5% from the bottom and 2.5% from the top.<br>● The median falls into either the upper or lower half of the data.<br>● Students sometimes relate the measures of central tendency with inappropriate measures of spread. For example, mean with quartiles and median with standard deviation | ● Trimmed mean is calculated by removing a percentage of data from the top and bottom of the data set. A 5% trimmed mean is calculated by removing 5% of the data at the bottom and 5% of the data at the top (actually removes 10% of the data).<br>● The median itself does not fall into either the upper or lower half of the data. However, in the case of an even number of data values, the two values used to compute the median are included in the lower and upper halves of the data, respectively.<br>● When using measures of center and spread, the standard deviation describes spread about the mean while quartiles describe spread about the median. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Measures of central tendency; mean, median and mode<br>● Trimmed mean<br>● Weighted average<br>● Measures of spread; range, variance and standard deviation<br>● Coefficient of variation<br>● Chebyshev's Theorem<br>● Percentiles | ● Compute the mean, median and mode of a given data set.<br>● Interpret the mean, median and mode of a data set.<br>● Find the trimmed mean for a given data set.<br>● Find the weighted average for a given data set.<br>● Compute the range, variance and | ● Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>● Students will be required to complete |

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| ● Quartiles<br>● Interquartile range (IQR)<br>● 5 number summary<br>● Box-and-whisker plots | standard deviation for a given data set.<br>● Interpret the range, variance and standard deviation of a given data set.<br>● Compute and interpret the coefficient of variation for a given data set.<br>● Apply Chebyshev's Theorem.<br>● Calculate and interpret percentiles including quartiles.<br>● Calculate and interpret the IQR.<br>● Calculate the five number summary for a given data set.<br>● Construct and interpret box-and-whisker plots.<br>● Use technology to perform statistical calculations and create statistical displays. | various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>● Students will evaluate their own understanding during homework reviews.<br>● Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>● Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>● Students will be assessed informally on a daily basis through student / teacher interactions.<br>● Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>● Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

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|---|---|---|
| ● Average<br>● Mode<br>● Median<br>● Mean<br>● Summation Symbol<br>● Sample Mean<br>● Population Mean<br>● Resistant Measure<br>● Trimmed Mean<br>● Distribution Shapes and Averages<br>● Weighted Average<br>● Harmonic Mean | ● Geometric Mean<br>● Range<br>● Variance<br>● Standard Deviation<br>● Sum of Squares<br>● Sample Variance<br>● Sample Standard Deviation<br>● Population Variance<br>● Population Standard Deviation<br>● Population Size<br>● Coefficient of Variation<br>● Chebyshev's Theorem | ● Outlier<br>● Mean of Grouped Data<br>● Standard Deviation of Grouped Data<br>● Moving Average<br>● Estimating Mean Using Stratified Samples<br>● Percentile<br>● Quartile<br>● Interquartile Range<br>● Five Number Summary<br>● Box-and-Whisker Plot<br>● Whiskers |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Agriculture
- Anthropology
- Archaeology

- Auto Insurance
- Business Administration
- Consumer
- Ecology
- Environmental Studies
- EPA
- Grades
- Health Care
- Investing
- Leisure
- Medical Science
- Merit Pay Scale
- Performance Rating
- Political Science
- Sociology
- Sports
- Wildlife

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**
Thomas A. Seltzer

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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 4** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**1 - 2 Weeks**<br>**(Approximately 10 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>**Unit 4 – Correlation and Regression**<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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| **Unit Title** | Unit 4 – Correlation and Regression |
| **Unit Summary** | This unit presents the concept of simple linear regression models and how they are used to make inferences. The strength of the linear relationship between two variables is measured using the correlation coefficient and the coefficient of determination is used to measure the proportion of variation in the response variable that is explained by the variation in the explanatory variable. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. How can you use a scatter diagram to visually estimate the degree of linear correlation of two random variables?<br>2. How do you compute the correlation coefficient and what does it tell you about the strength of the linear relationship between two random variables?<br>3. What is the least-squares criterion? How do you find the equation of the least squares line?<br>4. What is the coefficient of determination and what does it tell you about explained variation of y in a random sample of data pairs (x, y)? | 1. Construct scatter diagrams.<br>2. Estimate the "best fit" line visually.<br>3. Compute and interpret the correlation coefficient<br>4. State the least-squares criterion.<br>5. Find and graph the equation of the least-squares line.<br>6. Use the least-squares line to make predictions.<br>7. Explain the difference between interpolation and extrapolation.<br>8. Explain why extrapolation could lead to unreliable results.<br>9. Calculate and interpret the coefficient of determination. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.3** | Analyze linear models to make interpretations based on the data. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.3** | Analyze linear models to make interpretations based on the data. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● It is okay to switch the x and y values in a scatter plot.<br>● Regression lines with positive slopes can have negative correlations and vice versa<br>● When the correlation between an explanatory variable and a response variable is strong, it can be concluded that there is a cause-and-effect relationship between the variables<br>● An equation for predicting x-values can be derived from the LSRL by solving the equation for x.<br>● An r-value of zero implies that there is no relationship between x and y. | ● Although switching the x and y values in a scatter plot will yield the same correlation coefficient, it will result in a very different regression line. It is important to properly identify the explanatory and response variables before constructing a scatterplot.<br>● R-values for regression lines with positive slopes will be positive and r-values for lines with negative slopes will be negative.<br>● Strong correlation does not imply cause and effect.<br>● An equation for predicting x-values cannot be derived from the LSRL by solving the equation for x. You must recalculate the LSRL using x as the response variable and y as the explanatory variable.<br>● An r-value of zero implies that there is no *linear* relationship between x and y. There may be a different relationship between the variables (quadratic, cubic, etc.). |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Scatter diagrams<br>● Explanatory and Response variables<br>● Sample correlation coefficient<br>● Least-squares criterion<br>● Least-squares line<br>● Marginal change<br>● Influential points<br>● Residuals<br>● Interpolation and extrapolation<br>● Coefficient of determination | ● Construct and interpret scatter diagrams.<br>● Identify the explanatory and response variables in a data set.<br>● Calculate and interpret the sample correlation coefficient.<br>● State the least-squares criterion.<br>● Calculate and interpret the meaning of the LSRL.<br>● Explain the concept of marginal change.<br>● Explain the effects of influential points on the LSRL.<br>● Use the LSRL for prediction.<br>● Calculate and interpret the meaning of | ● Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>● Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>● Students will evaluate their own understanding during homework |

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| | residuals.<br>● Explain the process of interpolation and extrapolation and explain why extrapolation might yield unreliable results.<br>● Calculate and interpret the meaning of the coefficient of determination.<br>● Use technology to perform linear regressions. | reviews.<br>● Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>● Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>● Students will be assessed informally on a daily basis through student / teacher interactions.<br>● Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>● Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| ● Paired Data Values<br>● Scatter Diagram<br>● Explanatory Variable<br>● Response Variable<br>● No Linear Correlation<br>● Perfect Linear Correlation<br>● Positive Correlation<br>● Negative Correlation<br>● Sample Correlation Coefficient, $r$<br>● Population Correlation Coefficient, $\rho$ | ● Extrapolation<br>● Causation<br>● Lurking Variable<br>● Least-Squares Criterion<br>● Least-Squares Line, $\hat{y} = a + bx$<br>● Slope, $b$<br>● Intercept, $a$<br>● Meaning of Slope<br>● Marginal Change | ● Influential Point<br>● Residual<br>● Interpolation<br>● Extrapolation<br>● Coefficient of Determination, $r^2$<br>● Residual Plot<br>● Logarithmic Transformations<br>● Exponential Growth Model<br>● Power Law Model |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Archaeology
- Auto Accidents
- Crime
- Ecology
- Economics
- Education
- Geology
- Health Insurance
- Income
- Marketing

- Meteorology
- Ranching
- Research
- Sales
- Sociology
- Sports
- Veterinary Science

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**

Thomas A. Seltzer

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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 5** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**2 – 3 Weeks**<br>**(Approximately 15 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>**Unit 5 – Elementary Probability Theory**<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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|---|---|
| **Unit Title** | Unit 5 – Elementary Probability Theory |
| **Unit Summary** | This unit discusses the basics of probability including important rules associated with the calculations of probabilities such as the complement rule, the addition rule and the multiplication rule. Three main ways to determine the probability of an event are discussed including relative frequency, equally likely outcomes and intuition. Other topics include the law of large numbers, mutually exclusive events, and conditional probability. Finally tree diagrams and counting techniques are discussed as they related to finding probabilities. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. Why would anyone study probability?<br>2. What are the basic definitions and rules of probability<br>3. What are counting techniques, trees, permutations, and combinations? | 1. Assign probabilities to events.<br>2. Explain how the law of large numbers relates to relative frequencies.<br>3. Apply basic rules of probability.<br>4. Explain the relationship between statistics and probability.<br>5. Compute probabilities of compound events.<br>6. Compute probabilities involving independent or mutually exclusive events.<br>7. Use survey results to compute conditional probabilities.<br>8. Use tree diagrams to determine the outcomes in a sample |

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| | space.<br>9. Use permutations and combinations to compute outcomes.<br>10. Explain how counting techniques relate to probability in everyday life. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● A probability can be represented using any number..<br>● A high probability or low probability guarantees that an event will / will not occur.<br>● The rule for finding $P(A \text{ and } B)$ is always the same.<br>● When rolling two dice, the outcome (4, 3), four on the first die and 3 on the second die, and the outcome (3, 4), three on the first die and 4 on the second die, are the same.<br>● The rule for finding $P(A \text{ or } B)$ is always the same.<br>● Both the addition rule and the multiplication rule give the probability of two events occurring together.<br>● In both permutations and combinations, the order of occurrence is important. | ● Probabilities must be represented by numbers between 0 and 1.<br>● Just because the probability of an event is very high, it is not a certainty that the event will occur. Likewise, even though the probability of an event is very low, the event might still occur.<br>● The rule for finding $P(A \text{ and } B)$ depends on whether the events are independent or dependent.<br>● When tossing two dice, the outcomes (4, 3) and (3, 4) are different outcomes that both have the same probability of occurrence. When listing the sample space for rolling two dice, it is important to list both outcomes.<br>● The rule for calculating $P(A \text{ or } B)$ depends on whether or not the two events are mutually exclusive.<br>● The addition rule gives the probability that at least one of two specified events will occur. The multiplication rule gives the probability that two events will occur together.<br>● Only in permutations is the order important. In a combination, the order of occurrence is not considered. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Probability | ● Define probability. | ● Questioning and discussion will take |

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| ● Probability assignment<br>● Expressing probability results<br>● Law of Large Numbers<br>● Events, simple events, sample space<br>● The complement of an event<br>● Conditional probability<br>● Multiplication rules for probability<br>● Addition rules for probability<br>● Mutually exclusive events<br>● Contingency tables<br>● Multiplication rule of counting<br>● Tree diagrams<br>● Factorial notation<br>● Permutations<br>● Combinations | ● Explain how probability is assigned (intuition, relative frequency and using equally likely outcomes).<br>● Write probabilities with appropriate notation.<br>● State the law of large numbers.<br>● Identify events and sample spaces.<br>● Determine the complement of an event<br>● Interpret probabilities.<br>● Use multiplication rules to calculate probabilities.<br>● Calculate conditional probabilities.<br>● Interpret conditional probabilities.<br>● Use addition rules to calculate probabilities.<br>● Define and identify mutually exclusive events.<br>● Find probabilities using contingency tables.<br>● Use the multiplication rule of counting.<br>● Use tree diagrams to determine sample spaces and calculate probabilities.<br>● Determine the number of outcomes of an experiment using permutations and combinations.<br>● Interpret counting rules.<br>● Use technology to calculate permutations and combinations. | place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>● Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>● Students will evaluate their own understanding during homework reviews.<br>● Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>● Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>● Students will be assessed informally on a daily basis through student / teacher interactions.<br>● Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>● Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| ● Probability of an Event<br>● Intuition<br>● Relative Frequency<br>● Equally Likely Outcomes<br>● Law of Large Numbers<br>● Statistical Experiment<br>● Event<br>● Simple Event<br>● Sample Space<br>● Complement of an Event<br>● Independent Events | ● Dependent Events<br>● Probability of Event A and B<br>● Event A \| B<br>● Conditional Probability<br>● P (A \| B)<br>● Multiplication Rules of Probability<br>● More Than Two Independent Events<br>● Probability of Event A or B<br>● Event A and B<br>● Event A or B<br>● Mutually Exclusive Events | ● Addition Rules<br>● More Than Two Mutually Exclusive Events<br>● Contingency Table<br>● Basic Probability Rules<br>● Multiplication Rule of Counting<br>● Tree Diagram<br>● Factorial Notation<br>● Permutations Rule<br>● Combinations Rule |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.

- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Academics
- Agriculture
- Business
- Environmental
- Health Care
- Hiring
- Marketing
- Psychology
- Sales
- Scheduling
- Sports

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**

Thomas A. Seltzer

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|---|---|---|---|
| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 6** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**1 – 2 Weeks**<br>**(Approximately 14 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>**Unit 6 – The Binomial Probability Distribution and Related Topics**<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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|---|---|
| **Unit Title** | Unit 6 – The Binomial Probability Distribution and Related Topics |
| **Unit Summary** | This unit focuses on the concept of random variables. Explanations are given as to how to determine if a random variable is discrete or continuous. Methods for constructing the probability distribution of a discrete random variable are discussed and formulas for finding the mean and standard deviation of the discrete random variable are presented. Two special discrete probability distributions, the binomial distribution and the geometric distribution are also presented. |

**Unit Essential Questions:**

1. What is a random variable? How do you compute $\mu$ and $\sigma$ for a discrete random variable? How do you compute $\mu$ and $\sigma$ for linear combinations of independent random variables?
2. Many of life's experiences consist of some successes together with some failures. Suppose you make $n$ attempts to succeed at a certain project. How can you use the binomial probability distribution to compute the probability of $r$ successes?
3. How do you compute $\mu$ and $\sigma$ for the binomial

**Key Understandings:**

1. Distinguish between discrete and continuous random variables.
2. Construct and graph discrete probability distributions.
3. Compute the mean and standard deviation for a discrete probability distribution.
4. Compute the mean and standard deviation for a linear function of a random variable x.
5. Compute the mean and standard deviation for a linear combination of two independent random variables.
6. State the features of a binomial experiment.
7. Compute binomial probabilities.

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| distribution? | 8. Use the binomial table to find $P(r)$.<br>9. Use the binomial probability distribution to solve real-world problems.<br>10. Construct histograms for binomial distributions.<br>11. Compute the mean and standard deviation for a binomial distribution.<br>12. Compute the minimum number of trials $n$ needed to achieve a given probability of success $P(r)$. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● When asked to construct a discrete probability distribution, a graph of the distribution will suffice.<br>● The features of a binomial and a geometric experiment are the same. | ● A probability distribution consists of a table of values, the possible outcomes $x$ with their corresponding probability $P(x)$. A graph of a probability distribution is just that. It is not a probability distribution, it is the graph of a probability distribution.<br>● Binomial experiments have a fixed number of trials. Geometric experiments do not have a fixed number of trials. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Random Variables<br>● Discrete and Continuous Random Variables<br>● Probability distribution of a discrete random variable<br>● Mean and standard deviation of a discrete probability distribution<br>● Expected value<br>● Linear functions of a random variable | ● Define random variables.<br>● Distinguish between discrete and continuous random variables.<br>● Identify features of the probability distribution of a discrete random variable.<br>● Construct probability distributions for discrete random variables.<br>● Calculate the mean (expected value) | ● Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>● Students will be required to complete |

| | | |
|---|---|---|
| • Linear combinations of independent random variables<br>• Binomial experiment<br>• Binomial distribution formula<br>• Binomial distribution<br>• Mean and standard deviation of a binomial distribution<br>• Geometric distribution<br>• Mean and standard deviation of a geometric probability distribution | and standard deviation of a discrete probability distribution.<br>• Interpret discrete probability distributions.<br>• Calculate the mean, variance and standard deviation of linear functions and linear combinations of random variables.<br>• Identify features of a binomial experiment.<br>• Compute probabilities for a binomial experiment using the binomial distribution formula.<br>• Compute binomial probabilities using the probability distribution formula.<br>• Use a binomial distribution table to find binomial probabilities.<br>• Use technology to compute binomial probabilities.<br>• Interpret binomial probability distributions and their graphs.<br>• Graph binomial distributions.<br>• Calculate the mean and standard deviation of a binomial distribution.<br>• Find the mean and standard deviation of a geometric probability distribution. | various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>• Students will evaluate their own understanding during homework reviews.<br>• Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>• Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>• Students will be assessed informally on a daily basis through student / teacher interactions.<br>• Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>• Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| • Random Variable<br>• Discrete Random Variable<br>• Continuous Random Variable<br>• Probability Distribution<br>• Mean $\mu$ of a Probability Distribution<br>• Standard Deviation $\sigma$ of a Probability Distribution<br>• Expected Value, $\mu$<br>• Linear Function of a Random Variable | • Linear Combinations of Two Independent Random Variables<br>• Binomial Experiment<br>• Number of Trials, $n$<br>• Independent Trials<br>• Successes and Failures in a Binomial Experiment<br>• Probability of Success, $P(S) = p$<br>• Probability of Failure, $P(F) = q = 1 - p$ | • Number of Successes, $r$<br>• Binomial Probability Distribution, $P(r) = C_{n,r} p^r q^{n-r}$<br>• Binomial Coefficient, $C_{n,r}$<br>• Mean for the Binomial Distribution, $\mu = np$<br>• Standard Deviation for the Binomial Distribution, $\sigma = \sqrt{npq}$<br>• Geometric Probability Distribution |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Agriculture
- Aviation
- Business Ethics
- Criminal Justice

- Ecology
- Education
- Fishing
- Health Care
- History
- Insurance
- Law Enforcement
- Life Insurance
- Marketing
- Private Investigation
- Psychology
- Quality Control
- Sociology
- Sports

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**
Thomas A. Seltzer

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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 7 (Part I)** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**2 – 3 Weeks**<br>**(Approximately 16 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>**Unit 7 – Normal Curves and Sampling Distributions (Part I)**<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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|---|---|
| **Unit Title** | Unit 7 – Normal Curves and Sampling Distributions (Part I) |
| **Unit Summary** | This unit examines the properties and applications of the normal probability distribution. In the first part of this unit, basic information about normal distributions is discussed including what a normal distribution looks like, how it follows the empirical rule, and how to convert from raw data to z-scores and vice versa. The second part of this unit focuses on how normal distributions can be used to make inferences about a population parameter. |

**Unit Essential Questions:**

1. What are some characteristics of a normal distribution? What does the empirical rule tell you about data spread around the mean? How can this information be used in quality control?
2. Can you compare apples and oranges, or maybe elephants and butterflies? In most cases, the answer is no – unless you first standardize your measurements. What are a standard normal distribution and a standard z-score?
3. How do you convert any normal distribution to a standard normal distribution? How do you find probabilities of "standardized events?"

**Key Understandings:**

1. Graph a normal curve and summarize its important properties.
2. Apply the empirical rule to solve real world problems.
3. Use control limits to construct control charts. Examine the chart for three possible out-of-control signals.
4. Given $\mu$ and $\sigma$, convert raw data to z-scores.
5. Given $\mu$ and $\sigma$, convert z-scores to raw data.
6. Graph the standard normal distribution and find areas under the standard normal curve.
7. Compute the probability of "standardized events."
8. Find a z score from a given normal probability (inverse

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| | normal).<br>9. Use the inverse normal to solve guarantee problems. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● The empirical rule can be applied to all distributions.<br>● All normal curves are the same shape and size.<br>● Areas to the right of a z-score can be found by copying the value from a standard normal table.<br>● When drawing a sketch of a normal curve, it is only necessary to show the mean on the axis under the curve.<br>● We can always assume a given data set follows a normal distribution. | ● The empirical rule can only be applied to normal distributions whereas Chebyshev's theorem applies to all distributions.<br>● Not all normal curves are the same shape and size. This is due to the fact that the area under each normal curve must be equal to one and the standard deviation can change from one normal curve to another. In order to "standardize" a normal curve so it can be compared to other normal distributions, it is necessary to convert the curve to a standard normal curve. The standard normal curve always has a mean of zero and a standard deviation of one.<br>● It is important to note how a standard normal table is set up. Typically the values in the table represent areas under the normal curve to the left of a given z-score. In this case to find an area to the right of a z-score it is necessary to subtract the value of the area to the left of the z-score from 1.<br>● When drawing a normal curve, it is always advisable to mark three standard deviations to the left and three standard deviations to the right of the mean. This helps to correctly locate an x-value or a z-score on the curve.<br>● To determine if a data set is normal, several guidelines should be followed including making a histogram or box-and-whisker plot for the data, checking for outliers, checking for skewness and interpreting a normal probability plot. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Normal curves<br>● Empirical rule<br>● Normal distributions<br>● Z-scores and raw scores<br>● Standard normal distribution<br>● Inverse normal distribution<br>● Assessing normality | ● Draw and state the properties of a normal curve.<br>● State and apply the empirical rule.<br>● Interpret normal distributions.<br>● Convert from raw scores to z-scores.<br>● Convert from z-scores to raw scores.<br>● Interpret the meaning of z-scores.<br>● Graph and interpret the standard | ● Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology. |

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| | normal distribution.<br>● Use the standard normal distribution table to find areas under the standard normal curve.<br>● Find the areas under any normal curve.<br>● Use the inverse normal distribution to find z or x values that correspond to a given area under the normal curve.<br>● Follow a procedure for determining whether data have a normal distribution. | ● Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>● Students will evaluate their own understanding during homework reviews.<br>● Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>● Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>● Students will be assessed informally on a daily basis through student / teacher interactions.<br>● Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>● Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| ● Normal Distributions<br>● Normal Curves<br>● Downward Cup and Upward Cup on Normal Curves<br>● Inflection Points<br>● Symmetry of Normal Curves<br>● Normal Density Function | ● Empirical Rule<br>● Uniform Probability Distribution<br>● Z-Score<br>● Standard Units<br>● Raw Score<br>● Standard Normal Distribution<br>● Area Under the Standard Normal Curve | ● Left-Tail Style Table<br>● Areas Under Any Normal Curve<br>● Inverse Normal Distribution<br>● Normality Indicators<br>● Pearson's Index<br>● Normal Quantile Plot |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Archaeology
- Insurance
- Law Enforcement
- Laser Therapy
- Medical Science
- Physical Therapy

- Veterinary Science

**Additional Resources (May include, but not limited to the following):**
- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**
Thomas A. Seltzer

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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 7 (Part II)** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**1 – 2 Weeks**<br>**(Approximately 14 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>**Unit 7 – Normal Curves and Sampling Distributions (Part II)**<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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|---|---|
| **Unit Title** | Unit 7 – Normal Curves and Sampling Distributions (Part II) |
| **Unit Summary** | This unit examines the properties and applications of the normal probability distribution. In the first part of this unit, basic information about normal distributions is discussed including what a normal distribution looks like, how it follows the empirical rule, and how to convert from raw data to z-scores and vice versa. The second part of this unit focuses on how normal distributions can be used to make inferences about a population parameter. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. As humans, our experiences are finite and limited. Consequently, most of the important decisions in our lives are based on sample (incomplete) information. What is a probability sampling distribution? How will sampling distributions help us make good decisions based on incomplete information?<br>2. There is an old saying; All roads lead to Rome. In statistics, we could recast this saying: All probability distributions average out to be normal distributions (as the sample size increases). How can we take advantage of this in our study of sampling distributions? | 1. Define and correctly use statistical terms such as random sample, relative frequency, parameter, statistic and sampling distribution.<br>2. Use raw data to construct a relative frequency distribution for $\bar{x}$ values and compare the result to a theoretical sampling distribution.<br>3. For a normal distribution, use $\mu$ and $\sigma$ to construct the theoretical sampling distribution for the statistic $\bar{x}$.<br>4. For large samples, use sample estimates to construct a good approximate sampling distribution for the statistic $\bar{x}$. |

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| 3. The binomial and normal distributions are two of the most important probability distributions in statistics. Under certain limiting conditions, the binomial can be thought to evolve (or envelope) into the normal distribution. How can you apply this concept in the real world?<br>4. Many issues in life come down to success or failure. In most cases, we will not be successful all the time, so proportions of successes are very important. What is the probability sampling distribution for proportions? | 5. Explain the underlying meaning of the central limit theorem.<br>6. State the assumptions needed to use the normal approximation to the binomial distribution.<br>7. Compute $\mu$ and $\sigma$ for the normal approximation to the binomial distribution.<br>8. Use the continuity correction to convert a range of $r$ values to a corresponding range of $x$ values.<br>9. Convert the x-values to a range of standardized z-scores and find desired probabilities<br>10. Describe the sampling distribution for proportions $\hat{p}$. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● Samples and sampling distributions are essentially the same.<br>● If you do not know that a random variable x has a normal distribution, you can always conclude that the distribution of sample means is normal using the central limit theorem no matter the sample size.<br>● When using normal distributions to calculate probabilities, it is always necessary to make continuity corrections.<br>● The standard error for sampling distributions is always calculated the same way.<br>● A normal distribution should always be used to approximate a binomial distribution. | ● A sample refers to a single sample collected from a population whereas a sampling distribution involves all possible samples of a specific size that can be drawn from a population.<br>● In order to conclude a distribution of sample means is normal using the central limit theorem, the sample size must be at least 30.<br>● Continuity corrections are only necessary when using normal distributions to approximate binomial distributions since you are using a continuous distribution to approximate a discrete distribution.<br>● There are specific formulas for calculating the standard error depending on the type of sampling distribution being considered.<br>● There are certain conditions that dictate the use of the normal approximation to the binomial distribution. |

| Knowledge & Concepts | Skills & Competencies | Dispositions & Practices |
|---|---|---|
| • Statistic<br>• Parameter<br>• Statistical Inference<br>• Sampling distribution<br>• Sampling distribution of a sample mean<br>• Standard error<br>• The Central Limit Theorem<br>• Bias and Variability<br>• Normal Approximation to the Binomial Distribution.<br>• Sampling distribution of a sample proportion | • Distinguish between statistics and parameters.<br>• State the types of inferences we can make using estimation, testing and regression.<br>• Describe the information sampling distributions provide.<br>• State the characteristics of the sampling distribution of a sample mean.<br>• Calculate and explain what is meant by the standard error of a sampling distribution.<br>• Interpret the meaning of the central limit theorem.<br>• Use the central limit theorem to convert the sampling distribution of sample means to the standard normal distribution.<br>• Find probabilities regarding $\overline{x}$ .<br>• Explain what is meant by the bias and variability of a statistic.<br>• State the conditions for using the normal approximation to the binomial distribution.<br>• Show how to use a normal distribution to approximate a binomial distribution.<br>• Interpret the normal approximation to the binomial distribution.<br>• Explain what a continuity correction is and how it is calculated.<br>• State the characteristics of the sampling distribution of a sample proportion. | • Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>• Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>• Students will evaluate their own understanding during homework reviews.<br>• Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>• Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>• Students will be assessed informally on a daily basis through student / teacher interactions.<br>• Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>• Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| • Sample Statistic<br>• Population Parameter<br>• Sampling Distribution<br>• Mean of a Sampling Distribution of Sample Means, $\mu_{\overline{x}}$<br>• Standard Deviation of a Sampling Distribution of Sample Means, $\sigma_{\overline{x}}$ | • Standard Error of the Mean<br>• Central Limit Theorem<br>• Large Sample<br>• Normal Approximation to the Binomial Distribution<br>• Continuity Correction<br>• Sampling Distribution for a Sample Proportion, $\hat{p}$ | • Mean of the Sampling Distribution of Sample Proportions, $\mu_{\hat{p}}$<br>• Standard Deviation of the Sampling Distribution of Sample Proportions, $\sigma_{\hat{p}}$<br>• Standard Error of a Proportion |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.

- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Biology
- Business Administration
- Chemistry
- Crime
- Economics
- Engineering
- Finance
- Fishing
- Health
- Insurance
- Linguistics
- Medical Science
- Psychology
- Political Science
- Sociology
- Sports
- Wildlife

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**

Thomas A. Seltzer

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|---|---|---|---|
| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 8** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**1 – 2 Weeks**<br>**(Approximately 14 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>**Unit 8 – Estimation**<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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|---|---|
| **Unit Title** | Unit 8 – Estimation |
| **Unit Summary** | This unit explores the idea of using random samples to get information about a population. It discusses how to use both point estimates and confidence intervals to obtain this information. |

**Unit Essential Questions:**

1. How do you estimate the expected value of a random variable? What assumptions are needed? How much confidence should be placed in such estimates?
2. At the beginning design stage of a statistical project, how large a sample size should you plan to get?
3. What famous statistician worked for Guiness brewing company in Ireland? What has this got to do with constructing estimates from sample data?
4. How do you estimate the proportion *p* of successes in a binomial experiment? How does te normal approximation fit into this process?

**Key Understandings:**

1. Explain the meanings of confidence level, error of estimate, and critical value.
2. Find the critical value corresponding to a given confidence level.
3. Compute the confidence intervals for $\mu$ when $\sigma$ is known and interpret the results.
4. Compute the sample size to be used for estimating a mean $\mu$.
5. Use degrees of freedom and the Student's t distributions.
6. Find critical values using degrees of freedom and confidence levels.
7. Compute the confidence intervals for $\mu$ when $\sigma$ is unknown and interpret the results.

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| | 8. Compute the maximal margin of error for proportions using a given level of confidence.<br>9. Compute confidence intervals for $p$ and interpret the results.<br>10. Interpret poll results.<br>11. Compute the sample size to be used for estimating a proportion $p$ when we have an estimate for $p$.<br>12. Compute the sample size to be used for estimating a proportion $p$ when we have no estimate for $p$. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.7** | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● A confidence interval gives the probability that an interval contains a specific parameter.<br>● It does not matter if z-scores or t-scores are used when constructing a confidence interval.<br>● When given information for constructing a confidence interval, it is okay to perform the mathematical calculations before considering specific conditions. | ● Confidence intervals give boundaries between which we expect a parameter to fall. The percentage in a confidence interval should not be interpreted as a probability. It is an indicator of the percentage of intervals that would actually capture the parameter if multiple intervals were constructed.<br>● A decision must be made whether to use a z-score or a t-score when constructing a confidence interval. This decision is based on whether or not the standard deviation of the population is known and the size of the sample.<br>● There are several conditions that must be checked before constructing a confidence interval. It is important to check these conditions prior to constructing the confidence interval to ensure reliability. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Random variable, $x$<br>● Point estimate | ● State the assumptions about the random variable $x$. | ● Questioning and discussion will take place throughout the unit as concepts |

| | | |
|---|---|---|
| • Margin of error<br>• z Critical value<br>• C confidence interval for $\mu$<br>• Sample size for estimating $\mu$<br>• Student's t Distributions<br>• Degrees of freedom<br>• t critical value<br>• Inverse Cumulative Distribution Function<br>• Point estimates for $p$ and $q$<br>• C confidence interval for a proportion $p$<br>• Margin of error for polls<br>• Sample size for estimating $p$ | • Determine the point estimate for $\mu$.<br>• Find z critical values for confidence intervals.<br>• Construct a c confidence interval for $\mu$ when $\sigma$ is known.<br>• Interpret the meaning of confidence intervals.<br>• Determine the sample size needed to estimate the mean $\mu$.<br>• State the properties of the student's t distribution.<br>• Explain the meaning of degrees of freedom.<br>• Use a table to find t critical values for confidence intervals.<br>• Use the inverse cumulative distribution function to find t-values for given degrees of freedom and areas.<br>• Construct a c confidence interval for $\mu$ when $\sigma$ is unknown.<br>• Find point estimates for $p$ and $q$.<br>• Construct a c confidence interval for a proportion $p$.<br>• Determine the margin of error for polls.<br>• Determine the sample size needed to estimate a proportion, $p$.<br>• Use technology to construct confidence intervals. | are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>• Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>• Students will evaluate their own understanding during homework reviews.<br>• Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>• Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>• Students will be assessed informally on a daily basis through student / teacher interactions.<br>• Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>• Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| • Point Estimate for $\mu$<br>• Margin of Error<br>• Confidence Level, $c$<br>• Critical Values, $z_c$<br>• Maximal Margin of Error, $E$<br>• c Confidence Level | • Sample Size for Estimating $\mu$<br>• Student's t Distribution<br>• Degrees of Freedom<br>• Critical Values, $t_c$<br>• Point Estimate for $p$, $\hat{p}$ | • Confidence Interval for $p$<br>• Margin of Error for Polls<br>• Sample Size for Estimating $p$<br>• Plus Four Confidence Interval<br>• Two Confidence Intervals |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Agriculture
- Archaeology
- Auto Insurance
- Business
- Diagnostic Tests
- Ecology
- Finance
- Fishing
- Health Care
- Law Enforcement
- Lifestyle
- Marketing
- Medical Science
- Psychology
- Sports
- Wildlife
- Zoology

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**
Thomas A. Seltzer

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| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 9** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**3 Weeks**<br>**(Approximately 17 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>**Unit 9 – Hypothesis Testing**<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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|---|---|
| **Unit Title** | Unit 9 – Hypothesis Testing |
| **Unit Summary** | Hypothesis testing is a major component of inferential statistics. In hypothesis testing, we propose a specific value for the population parameter in question. Then we use sample data from a random sample and probability to determine whether or not to reject this specific value for the parameter. This unit deals with the processes involved in performing hypothesis tests. |

**Unit Essential Questions:**

1. Many of life's questions require a yes or no answer. When you must act on incomplete (sample) information, how do you decide whether to accept or reject a proposal?
2. What is the P-value of a statistical test? What does this measurement have to do with performance reliability?
3. How do you construct statistical tests for $\mu$? Does it make a difference whether $\sigma$ is known or unknown?
4. How do you construct statistical tests for the proportion $p$ of successes in a binomial experiment?

**Key Understandings:**

1. Understand the rationale for statistical tests.
2. Identify the null and alternate hypotheses in a statistical test.
3. Identify right-tailed, left-tailed, and two-tailed tests.
4. Use a test statistic to compute a P-value.
5. Recognize types of errors, level of significance, and power of a test.
6. Understand the meaning and risks of rejecting or failing to reject the null hypothesis.
7. Know the general procedure for testing using P-values.
8. Test $\mu$ when $\sigma$ is known using the normal distribution.
9. Test $\mu$ when $\sigma$ is unknown using a Student's t distribution.
10. Understand the "traditional" method of testing that uses

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| | critical regions and critical values instead of P-values.<br>11. Identify the components needed for testing a proportion.<br>12. Compute the sample test statistic.<br>13. Find the P-value and conclude the test. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● Statistics can be used to state null and alternative hypotheses.<br>● When stating hypotheses, both the null and alternative hypotheses can be stated as statements of equality.<br>● When you perform a hypothesis test you can support the null hypothesis.<br>● A hypothesis test can be used to prove a claim.<br>● The P-value is the probability of a type I error.<br>● Z-scores can be used in a hypothesis test when the sample standard deviation is known.<br>● P-values are used to determine critical regions.<br>● Hypotheses tests for $\mu$ and $p$ are identical. | ● Null and alternative hypotheses must always be stated using parameters.<br>● Only the null hypothesis is stated as a statement of equality and it must be stated is such. The alternative hypothesis is stated as an inequality using $>, <$ or $\neq$.<br>● You can only support the alternative hypothesis. If you are unable to support the alternative hypothesis, this does not imply support for the null hypothesis. It simply means there is not enough evidence to decide against it.<br>● Hypotheses tests do not prove anything. They can only give strong evidence to support the alternative (this does not prove that the alternative is true).<br>● The P-value does not represent the probability of a type I error. The probability of a type I error is represented by $\alpha$, the level of significance of the test.<br>● Z-scores should only be used in a hypothesis test when the standard deviation of the population is known and we can assume a noral distribution or a large sample size $(n \geq 30)$.<br>● Critical regions are determined by considering the level of significance and whether the test is right-, left- or two-tailed.<br>● It is important to determine which type of hypothesis test you are performing (test for mean or test for proportion) as the assumptions and test statistics for each type of test differ. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Null and Alternative hypotheses | ● Correctly state null and alternative | ● Questioning and discussion will take |

| | | |
|---|---|---|
| • Types of tests; left-tailed, right tailed and two tailed<br>• Hypothesis tests for $\mu$ when $\sigma$ is known and when $\sigma$ is unknown<br>• Test statistic<br>• Assumptions of a test<br>• P-values<br>• Types of errors<br>• Concluding a hypothesis test<br>• Power of a test<br>• Critical regions<br>• Hypothesis tests for p | hypotheses.<br>• Identify whether a test is left-tailed, right-tailed or two tailed using the alternative hypothesis.<br>• State the assumptions of a test.<br>• Calculate the test statistic for a test.<br>• Calculate p-values.<br>• Correctly state the conclusion of a hypothesis test.<br>• Perform hypotheses tests for $\mu$ showing all relevant steps and interpret the results.<br>• Identify type I and type II errors and evaluate their significance.<br>• Explain what is meant by the power of a test.<br>• Use critical values instead of p-values to perform hypothesis tests.<br>• Perform hypothesis tests for p showing all relevant steps and interpret the results. | place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>• Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>• Students will evaluate their own understanding during homework reviews.<br>• Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>• Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>• Students will be assessed informally on a daily basis through student / teacher interactions.<br>• Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>• Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| • Hypothesis Testing<br>• Hypothesis<br>• Null Hypothesis<br>• Alternate Hypothesis<br>• Right-Tailed Test<br>• Two-Tailed Test<br>• Left-Tailed Test<br>• Sample Test Statistic | • P-value<br>• Type I Error<br>• Type II Error<br>• Level of Significance of a Test<br>• Probability of Type I Error, $\alpha$<br>• Probability of Type II Error, $\beta$<br>• Power of a Test, $1-\beta$ | • Statistical Significance<br>• Fail to Reject the Null, $H_0$<br>• Reject the Null, $H_a$<br>• Degrees of Freedom for testing $\mu$ when $\sigma$ is unknown<br>• Critical Region Method<br>• Critical Value |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Agriculture
- Archaeology
- Athletics
- Civil Service
- Ecology
- Finance
- Fishing
- Insurance
- Investing
- Marketing
- Medical Science
- Meteorology
- Physics
- Sociology
- Veterinary Science
- Wildlife

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**
Thomas A. Seltzer

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|---|---|---|---|
| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 10** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**3 Weeks**<br>**(Approximately 17 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>**Unit 10 – Inferences about Differences**<br>Unit 11 – Additional Topics Using Inference (Part I)<br>Unit 11 – Additional Topics Using Inference (Part II) |

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|---|---|
| **Unit Title** | Unit 10 – Inferences about Differences |
| **Unit Summary** | In this unit the discussion of hypothesis testing and confidence intervals continues. Many statistical applications involve the use of paired data or the comparison of two population means or proportions. This unit specifically looks at inferences involving paired differences, differences of means, and differences of proportions. |

**Unit Essential Questions:**

1. What are the statistical advantages of paired data values? How do we construct statistical tests?
2. How do we compare means from two independent populations when we know $\sigma$ for each population?
3. What if we want to compare means from two independent populations, but we do not know $\sigma$ for each population?
4. How do we use sample data to compare proportions from two independent populations?

**Key Understandings:**

1. Identify paired data and dependent samples.
2. Explain the advantages of paired data tests.
3. Compute differences and the sample test statistic for tests involving paired differences.
4. Estimate the P-value and conclude the test for tests involving paired differences.
5. Identify independent samples and sampling distributions.
6. Compute the sample test statistic and P-value for testing the difference between two means, $\mu_1 - \mu_2$.
7. Find confidence intervals for the difference between two means, $\mu_1 - \mu_2$.

| | |
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| | 8. Compute the sample test statistic and P-value for testing the difference between two population proportions, $p_1 - p_2$.<br>9. Find confidence intervals for the difference between two population proportions, $p_1 - p_2$. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● Test involving paired differences and tests about the difference of two means are the same. | ● Tests involving paired differences deal with dependent samples while tests involving the differences between two means deal with independent samples. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Paired data<br>● Mean and standard deviation of differences from data pairs<br>● Hypothesis test for paired data<br>● Independent samples<br>● Hypothesis tests for the difference between two means<br>● Confidence intervals for the difference between two means<br>● Critical regions<br>● Hypothesis test for the difference between two proportions<br>● Confidence intervals for the difference between two proportions | ● Identify paired data sets.<br>● Find the mean and standard deviation of the differences from data pairs.<br>● Perform hypothesis tests for paired data.<br>● Distinguish between independent and dependent samples.<br>● Perform hypothesis tests for differences between two means $\mu_1 - \mu_2$ when $\sigma_1 \text{ and } \sigma_2$ are known.<br>● Perform hypothesis tests for differences between two means | ● Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>● Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>● Students will evaluate their own |

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| | $\mu_1 - \mu_2$ when $\sigma_1 \text{ and } \sigma_2$ are unknown.<br>● Calculate confidence intervals for differences between two means $\mu_1 - \mu_2$ when $\sigma_1 \text{ and } \sigma_2$ are known..<br>● Calculate confidence intervals for differences between two means $\mu_1 - \mu_2$ when $\sigma_1 \text{ and } \sigma_2$ are unknown.<br>● Use critical regions when performing hypothesis tests.<br>● Perform hypothesis tests for the difference between two proportions.<br>● Calculate confidence intervals for the differences between two proportions.<br>● Use technology to perform hypothesis tests and to construct confidence intervals. | understanding during homework reviews.<br>● Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>● Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>● Students will be assessed informally on a daily basis through student / teacher interactions.<br>● Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>● Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| ● Paired Data<br>● Creating data pairs<br>● Mean of differences from data pairs, $\mu_d$.<br>● Standard deviation of differences from data pairs, $s_d$. | ● Dependent samples<br>● Independent samples<br>● Degrees of freedom for $\mu_1 - \mu_2$ distribution when $\sigma_1 \text{ and } \sigma_2$ are unknown. | ● Satterthwaite's approximation for degrees of freedom Pooled standard deviation<br>● Pooled estimate of proportion, $\bar{p}$.<br>● Criteria for using the normal approximation to the binomial. |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Archaeology
- Agriculture
- Business
- Crime
- Demographics
- Ecology
- Economics
- Education
- Environment
- Fishing
- Funding
- Management
- Marketing

- Medical Science
- Medicine
- Political Science
- Sociology
- Sports
- Wildlife

**Additional Resources (May include, but not limited to the following):**
- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**
Thomas A. Seltzer

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|---|---|---|---|
| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 11 (Part I)** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**1 – 2 Weeks**<br>**(Approximately 13 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>**Unit 11 – Additional Topics Using Inference (Part I)**<br>Unit 11 – Additional Topics Using Inference (Part II) |

| | |
|---|---|
| **Unit Title** | Unit 11 – Additional Topics Using Inference (Part I) |
| **Unit Summary** | The first part of this unit introduces applications of the chi-square probability distribution.. Specifically this chapter addresses the properties of the chi-square distribution, how to perform test for independence or homogeneity and goodness-of-fit. Tests of variance are also presented. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. How do you decide if random variables are dependent or independent?<br>2. How do you decide if different populations share the same proportions of specified characteristics?<br>3. How do you decide if two distributions are not only dependent but actually the same distribution?<br>4. How do you compute the confidence intervals and tests for $\sigma$. | 1. Set up a test to investigate independence of random variables.<br>2. Use contingency tables to compute the sample $\chi^2$ statistic.<br>3. Find or estimate the p-value of the sample $\chi^2$ statistic and complete the test for independence.<br>4. Conduct a test of homogeneity of populations.<br>5. Set ups a test to investigate how well a sample distribution fits a given distribution.<br>6. Use observed and expected frequencies to compute the sample $\chi^2$ statistic.<br>7. Find or estimate the P-value and complete the goodness of |

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| | fit test.<br>8. Set up a test for a single variance, $\sigma^2$.<br>9. Compute the sample $\chi^2$ statistic.<br>10. Use the $\chi^2$ distribution to estimate a P-value and conclude the test for $\sigma^2$. |

| **Focus Standards Addressed in the Unit:** | |
|---|---|
| ***Standard Number*** | *Standard Description* |
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.3.HS.B.1** | Summarize, represent, and interpret data on a single count or measurement variable. |
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| ● Degrees of freedom for the Chi square test for independence and the Chi square goodness of fit test are calculated the same way.<br>● There is no difference between the Chi square test for independence and the Chi square test for homogeneity. | ● To find the number of degrees of freedom for the Chi square test for independence, use the following formula: d.f. = (R-1)(C-1) where R is the number of rows in a contingency table and C is the number of columns in a contingency table. To find the number of degrees of freedom for the Chi square goodness of fit test, use d.f. = k – 1 where k represents the number of categories.<br>● Though the process for conducting the Chi square test of independence and the Chi square test of homogeneity are the same, there are two main differences in the set up of the tests, namely the sampling method and the hypotheses. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| ● Chi-Square distribution<br>● Contingency Table<br>● Observed Frequency | ● Use the Chi-Square distribution.<br>● Determine the size of a contingency table. | ● Questioning and discussion will take place throughout the unit as concepts are presented and during homework |

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|---|---|---|
| • Expected Frequency<br>• Degrees of freedom<br>• Chi-Square Test of Independence<br>• Chi-Square Test of Homogeneity<br>• Chi-Square Goodness-of-Fit test<br>• Testing $\sigma^2$ | • Calculate expected frequencies.<br>• Determine the degrees of freedom for the Chi square test of independence and the Chi square test of homogeneity.<br>• Perform hypothesis tests for independence of two statistical variables.<br>• Perform hypothesis tests for homogeneity.<br>• Differentiate between tests of independence and tests of homogeneity.<br>• Determine the degrees of freedom for the chi-square goodness of fit test.<br>• Perform the chi-square goodness of fit test and interpret the results.<br>• Perform hypothesis tests for $\sigma^2$. | reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>• Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>• Students will evaluate their own understanding during homework reviews.<br>• Students will receive an assignment sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>• Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>• Students will be assessed informally on a daily basis through student / teacher interactions.<br>• Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>• Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| • Chi-square distribution<br>• Test of independence<br>• Hypotheses, tests of independence<br>• Observed frequency of a cell<br>• Contingency table | • Expected frequency of a cell<br>• Row total<br>• Column total<br>• Sample test statistic, $\chi^2$<br>• Degrees of freedom for $\chi^2$ distribution and tests of independence | • Test of homogeneity<br>• Goodness-of-fit test<br>• Degrees of freedom for $\chi^2$ distribution and goodness-of-fit tests<br>• Hypotheses tests about the variance $\sigma^2$ |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Accounting
- Archaeology
- Census
- Ecology
- Education
- Engineering
- Law
- Marketing
- Medical Science
- Meteorology
- Politics
- Psychology
- Sociology
- Veterinary Science

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**

Thomas A. Seltzer

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|---|---|---|---|
| | | | **Course: Statistics**<br>**Grade Level: 11 / 12**<br>**Unit: 11 (Part II)** |
| **Course/Subject:**<br>**Statistics** | **Grade:**<br>**11 / 12** | | **Suggested Timeline:**<br>**1 Week**<br>**(Approximately 6 Days)** |

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| **Grade Level Summary** | **Statistics** is a year-long course designed to provide students with a basic knowledge of important statistical concepts and applications which are addressed in the Pennsylvania Core Standards for mathematics. This course is for students who have an interest in developing skills in the area of statistics but are not interested in studying statistics at the advanced placement (AP) level. The course will address topics in both descriptive and inferential statistics. Topics will include: organizing data, measures of center and variation, correlation and regression, probability and probability distributions, sampling distributions, estimation, and inferences using hypothesis tests. |
| **Grade Level Units** | Unit 1 - Getting Started<br>Unit 2 - Organizing Data<br>Unit 3 – Averages and Variation<br>Unit 4 – Correlation and Regression<br>Unit 5 – Elementary Probability Theory<br>Unit 6 – The Binomial Probability Distribution and Related Topics<br>Unit 7 – Normal Curves and Sampling Distributions (Part I)<br>Unit 7 – Normal Curves and Sampling Distributions (Part II)<br>Unit 8 – Estimation<br>Unit 9 – Hypothesis Testing<br>Unit 10 – Inferences about Differences<br>Unit 11 – Additional Topics Using Inference (Part I)<br>**Unit 11 – Additional Topics Using Inference (Part II)** |

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|---|---|
| **Unit Title** | Unit 11 – Additional Topics Using Inference (Part II) |
| **Unit Summary** | The second part of this unit discusses testing the correlation coefficient $\rho$, testing the slope $\beta$ of the least-squares line, as well as finding confidence intervals for predicted values based on a specific value for x and the least squares line, and finding confidence intervals for the slope $\beta$ of the least squares line. |

| **Unit Essential Questions:** | **Key Understandings:** |
|---|---|
| 1. How do you test a correlation coefficient<br>2. What is the standard error of estimate? How do you compute it and where is it used?<br>3. How do you compute confidence intervals for a least-squares prediction?<br>4. The slope of the least-squares line represents rate of growth. How can you determine if the rate of growth is statistically significant? | 1. Test the correlation coefficient $\rho$.<br>2. Use sample data to compute the standard error of estimate $S_e$.<br>3. Test the slope $\beta$ of the least-squares line.<br>4. Find a confidence interval for the value of y predicted for a specified value of x. |

**Focus Standards Addressed in the Unit:**

| *Standard Number* | *Standard Description* |
|---|---|
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.3** | Analyze linear models to make interpretations based on the data. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |

| **Important Standards Addressed in the Unit:** | |
|---|---|
| **CC.2.4.HS.B.2** | Summarize, represent, and interpret data on two categorical and quantitative variables. |
| **CC.2.4.HS.B.3** | Analyze linear models to make interpretations based on the data. |
| **CC.2.4.HS.B.4** | Recognize and evaluate random processes underlying statistical experiments. |
| **CC.2.4.HS.B.5** | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |
| **CC.2.4.HS.B.6** | Use the concepts of independence and conditional probability to interpret data. |

| **Misconceptions:** | **Proper Conceptions:** |
|---|---|
| • Showing that x and y are either positively or negatively correlated implies that x causes y.<br>• When examining the correlation between two variables, it is enough to simply look at a scatterplot. | • Correlation does not imply causation. Correlation simply shows that there is some type of relationship between the two variables. It does not show that one variable causes the other.<br>• When examining the correlation between two variables, it is important to consider the spread of the points about the least-squares line. This can be measured using the standard error of estimate, the coefficient of correlation or the coefficient of determination. |

| **Knowledge & Concepts** | **Skills & Competencies** | **Dispositions & Practices** |
|---|---|---|
| • Population correlation coefficient $\rho$<br>• Requirements for inferences concerning linear regression<br>• Hypothesis test for $\rho$<br>• Standard error of estimate<br>• Population slope of least squares line, $\beta$<br>• Standard error for b<br>• Hypothesis test for $\beta$<br>• Confidence interval for y | • State the requirements for statistical inference for the population correlation coefficient $\rho$ and the slope $\beta$ of the population least-squares line.<br>• Perform hypothesis tests for $\rho$ and interpret the results.<br>• Compute the standard error of estimate.<br>• Calculate the standard error for b.<br>• Perform hypothesis tests for $\beta$ and interpret the results.<br>• Construct a confidence interval for a predicted y from the least squares line | • Questioning and discussion will take place throughout the unit as concepts are presented and during homework reviews with the goal of improving / reinforcing student understanding of concepts and encouraging students to communicate effectively using statistical terminology.<br>• Students will be required to complete various activities and assignments which will require students to think critically and may involve collaborative efforts.<br>• Students will evaluate their own understanding during homework reviews.<br>• Students will receive an assignment |

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| | | sheet for the unit that will communicate the timeline for the unit as well as the assignments and assessments to be completed.<br>● Students will be required to take notes using a skeletal note packet which will serve as a study guide.<br>● Students will be assessed informally on a daily basis through student / teacher interactions.<br>● Summative assessments will include quiz(zes), test(s) and project(s) which will provide students with feedback concerning their understanding of the concepts.<br>● Pacing of lessons may be adjusted and additional examples may be provided as necessary to enhance student understandings and competencies. |

**Academic Vocabulary:**

| | | |
|---|---|---|
| ● Population correlation coefficient $\rho$<br>● Standard error of estimate $S_e$ | ● Population slope of the least-squares line $\beta$<br>● C Confidence level for y | ● Confidence prediction band<br>● Serial correlation<br>● Confidence interval for $\beta$ |

**Evidence: Assessments and Performance Task(s)**

- Homework – Problems will be assigned and reviewed daily to reinforce and enhance students' understanding of concepts.
- Unit Quizzes – Quizzes will be given throughout the unit to evaluate students' understanding of the material.
- Unit Test – A unit test will be given at the end of the unit to evaluate students' overall understanding of the unit.
- Unit Notebook – Students' notebooks will be checked at the end of each instructional unit to reinforce organizational skills and to provide additional accountability for completion of class assignments and homework.
- Statistical Project(s) – Projects may be assigned to provide additional opportunities for student assessment.

**Interdisciplinary Connections:**

- Ecology
- Oceanography
- Physiology
- Sports

**Additional Resources (May include, but not limited to the following):**

- Textbook Ancillary Materials
- Online resources
- Teacher generated resources

**Created By:**
Thomas A. Seltzer