# Academic Trigonometry & Advanced Math / 11 -12

## Prerequisites: Fundamental Concepts of Algebra

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | P – Prerequisites: Fundamental Concepts of Algebra | 15 Days |

### Grade Level Units

Trigonometry is a rich mathematical content area that blends geometric, graphical, and algebraic reasoning. Students learn a variety of techniques for solving triangles, manipulating trigonometric expressions, and graphing functions. The course begins by laying the foundation for trigonometry by keying in on the fundamental concepts of algebra. There is a heavy emphasis on solving, graphing, and analyzing functions. Throughout the course, it becomes increasingly important for students to explore, understand, and investigate various forms of functions. From linear, quadratic, rational, inverse, polynomial, exponential, logarithmic, to trigonometric functions, students will be learn to analyze and discuss the impact that transformations have on the domain and range of these functions. This prepares students for the culminating themes associated with trigonometry including degree and radian measures, right triangle trigonometry, graphing the six trigonometric functions, applications, inverse, and finally, analytic trigonometry where students use trigonometric identities to verify identities and ultimately solve trigonometric equations. Additional topics in trigonometry include law of sines, cosines, polar coordinates, and vectors. Beyond the trigonometric units, the course shifts to more advanced mathematics topics including extending solving systems of equations into three variables using various methods including matrices. Systems of linear inequalities and linear programming are included in this content. The final unit of study involves higher-level geometric analysis of conic sections followed by a statistics unit that includes sequences and series. Many trigonometric topics serve as introductions into branches of mathematics typically reserved for collegiate study in areas such as Statistics, Linear Algebra, and Calculus. For students extending their mathematical studies to a collegiate level while in high school, this course develops skills which provide opportunity for understanding the theoretical underpinnings of calculus.

### Grade Level Summary

**Unit P - Prerequisites: Fundamental Concepts of Algebra**

Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

### Unit Title

Prerequisites: Fundamental Concepts of Algebra

### Unit Overview

This unit reviews fundamental concepts of algebra that are prerequisites for the study of college algebra. Throughout the unit, students see how the special language of algebra describes their world.

### Unit Essential Questions

1. How is the intersection or union of sets determined?
2. What are the rules for exponents?
3. What is the process for rationalizing the denominator?
4. What is the process for identifying the degree of a polynomial?
5. How are polynomials factored?
6. How are rational expressions simplified?

### Key Understandings

1. Finding the intersection/union of sets
2. Simplifying algebraic expressions including absolute value
3. Applying the laws of exponents including rational exponents
4. Writing numbers in scientific notation
5. Performing operations with radicals including simplifying and rationalizing the denominator
6. Adding, subtracting, multiplying, dividing, factoring and simplifying polynomials

| | |
|---|---|
| | 7. Adding, subtracting, multiplying, dividing, simplifying and finding the domain of rational expressions |

| **Focus Standards Addressed in the Unit** | |
|---|---|
| CC.2.1.HS.F.1 | Apply and extend properties of exponents to solve problems with rational exponents. |
| CC.2.1.HS.F.2 | Apply properties of rational and irrational numbers to solve real world or mathematical problems. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.1.HS.F.3 | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| CC.2.1.HS.F.5 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
| CC.2.2.HS.D.2 | Write expressions in equivalent forms to solve problems. |
| CC.2.2.HS.D.7 | Create and graph equations or inequalities to describe numbers or relationships. |
| CC.2.2.HS.D.9 | Use reasoning to solve equations and justify the solution method. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. Confusing union and intersection of sets<br>2. Taking the absolute value of an expression before simplifying what is inside the absolute value bars<br>3. Thinking a negative exponent has anything to do with the sign of the answer<br>4. Simplifying a negative number in parenthesis to an even power as a negative solution<br>5. Not multiplying radicals that have like indexes but unlike radicands<br>6. Forgetting to use the conjugate to rationalize<br>7. Not looking throughout the whole problem when finding the restrictions on the denominator of a fraction, instead only looking at final solution | 1. Union of sets includes all elements of the sets. Intersection of sets includes only the like elements of the sets.<br>2. Always follow order of operations and simplify what is in the grouping symbols before taking the absolute value.<br>3. When you see a negative exponent think fraction. The negative exponent changes the location of the base.<br>4. A negative number in parenthesis raised to an even power is always a positive.<br>5. As long as the indexes are the same, you can multiply any radicals together no matter what the radicands are.<br>6. Rationalize a denominator by multiplying both the numerator and denominator of the fraction by the conjugate of the denominator.<br>7. The restrictions on the domain of a rational expression include any values that would make the rational expression undefined, that is, any value that creates a zero in the denominator of the fraction. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • Algebraic Expressions, Mathematical Models, and Real Numbers<br>• Exponents and Scientific Notation<br>• Radicals and Rational Exponents<br>• Polynomials<br>• Factoring Polynomials<br>• Rational Expressions | • Determine the intersection and union of sets.<br>• Simplify algebraic expressions using the rules of exponents.<br>• Simplify radicals including rationalizing the denominator.<br>• Perform operations with polynomials.<br>• Factor polynomials.<br>• Simplify rational expressions. | • Order of Operations<br>• Union<br>• Intersection<br>• Absolute Value<br>• Algebraic Expression<br>• Distributive Property<br>• Exponent Rules<br>• Scientific Notation<br>• Rules of Radicals<br>• Rationalizing the Denominator<br>• Conjugates<br>• Rational Exponents<br>• Roots<br>• Reducing the Index |

| | | |
|---|---|---|
| | | • Degree<br>• Leading Coeeficient<br>• FOIL<br>• Factor<br>• GCF<br>• Sum/Difference of Cubes<br>• Rational Expression<br>• Domain<br>• LCD |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which connects prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Expenses & Debt
- Exercise & Health
- Time Dilation
- Ethnic Diversity
- Gender Balance

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Equations & Inequalities

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Equations & Inequalities | 18 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
**Unit 1 – Equations & Inequalities**
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Equations & Inequalities

**Unit Overview**

Formulas can be used to explain what is happening in the present and to make predictions about what might occur in the future. In this unit, students will learn to use formulas in new ways that will help them to recognize patterns, logic, and order in a world that can appear chaotic to the untrained eye.

**Unit Essential Questions**

1. How are the intercepts of an equation determined?
2. How do you solve linear and rational equations?
3. What is the five step plan for solving word problems?
4. How are operations with complex numbers performed?
5. What methods can be used to solve quadratic equations?
6. How are radical equations and equations with rational exponents solved?
7. What is interval notation and how is it used?

**Key Understandings**

1. Graph functions using a table of values.
2. Verify graphs of functions using the graphing calculator.
3. Find the intercepts of a function.
4. Solve linear and rational equations.
5. Find the restrictions of rational equations.
6. Solve word problems using models and applications.
7. Perform operations with complex numbers.
8. Solve quadratic equations by graphing, factoring, quadratic formula, completing the square, and using the square root property
9. Solve and check solutions to radical equations.
10. Solve equations with rational exponents.
11. Solve equations using u-substitution.
12. Solve absolute value equations.
13. Solve and graph linear inequalities including combined inequalities.
14. Solve and graph absolute value inequalities.

**Focus Standards Addressed in the Unit**

| | |
|---|---|
| CC.2.2.HS.D.7 | Create and graph equations or inequalities to describe numbers or relationships. |
| CC.2.2.HS.D.10 | Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. |

| Important Standards Addressed in the Unit | |
|---|---|
| CC.2.1.HS.F.3 | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| CC.2.1.HS.F.5 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
| CC.2.2.HS.D.2 | Write expressions in equivalent forms to solve problems. |
| CC.2.2.HS.D.9 | Use reasoning to solve equations and justify the solution method. |
| | |

**Misconceptions**

1. The solution obtained from solving a rational equation for a variable is a solution to the equation.
2. Confusing an inconsistent equation with an identity.
3. A complex conjugate has the same sign in front of the complex part of the complex number you are trying to rationalize.
4. Square rooting both sides of an equation produces only one solution.
5. All solutions obtained by solving radical equations for a variable are solutions to the radical equation.
6. Confusing conjunctions and disjunctions.

**Proper Conceptions**

1. You need to check solutions to rational equations to make sure the solution you obtain is not a restriction on the domain of the function.
2. An inconsistent equation has no solution whereas an identity is an equation where the solution is infinite.
3. A complex conjugate has the opposite sign in front of the complex part of the complex number you are trying to rationalize.
4. When square rooting both sides of an equation you must consider both the positive and negative of the square root solutions.
5. You must check solutions to radical equations to make sure that the solutions satisfy the original problem.
6. Conjunctions are the "and" problems where the solution is the intersection of the two inequalities. Disjunctions are the "or" problems where the solution is everything you are given.

**Concepts**

- Graphs and Graphing Utilities
- Linear Equations and Rational Equations
- Models and Applications
- Complex Numbers
- Quadratic Equations
- Other Types of Equations
- Linear Inequalities and Absolute Value Inequalities

**Competencies**

- Graph functions using a table of values and the graphing calculator.
- Solve linear and rational equations.
- Solve word problems using the five step plan.
- Perform operations with complex numbers.
- Solve quadratic equations.
- Solve radical equations and equation with rational exponents.
- Solve linear and absolute value inequalities.

**Vocabulary**

- Domain
- Range
- Intercepts
- Window
- Linear
- Rational
- Restriction
- Empty Set
- Conditional Statement
- Identity
- Inconsistent
- Five Step Plan
- Complex Numbers
- Imaginary Numbers
- Complex Conjugate
- Rationalize
- Standard Form
- Parabola
- Factoring
- Graphing
- Quadratic Formula
- Completing the Square
- Square Root Property
- Zeros
- Roots
- "u" Substitution
- Radical

| | | |
|---|---|---|
| | | • Rational Exponent<br>• Exponent over Index<br>• Absolute Value<br>• Interval Notation<br>• Set-Builder Notation<br>• Combined Inequality<br>• Conjunction<br>• Disjunction |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
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**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Meteorology
- Expenses & Debt
- Health and Wellness
- Teaching & Learning
- Chemistry
- Sports

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Functions & Graphs

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Functions & Graphs | 19 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
**Unit 2 – Functions & Graphs**
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Functions & Graphs

**Unit Overview**

In this unit, students learn to approach situations mathematically by creating formulas, called functions, that model data over time. Understanding the concept of a function will give students a new perspective on many situations, ranging from global warming to using mathematics in a way similar to making a movie.

**Unit Essential Questions**

1. What is the vertical line test and how is it used?
2. What does it mean for a function to be even or odd?
3. How is the slope of a line determined?
4. How do you find the equation for the line of best fit?
5. How is slope used to determine if two lines are parallel or perpendicular?
6. How are transformations applied to functions?
7. How is the domain of a composite function determined?
8. What is the process for finding an inverse function?
9. How is the distance and midpoint between two ordered pairs determined?

**Key Understandings**

1. Determining if a relation is a function using the vertical line test.
2. Evaluating functions over a given domain.
3. Graphing two functions together on a graph and describing how the two graphs are related.
4. Finding the difference quotient.
5. Evaluating piecewise functions.
6. Finding where functions are increasing, decreasing, and/or constant
7. Finding the relative maximum and/or relative minimum of a function.
8. Determining if a function is even, odd, or neither.
9. Evaluating a greatest integer function.
10. Finding the slope of a line.
11. Writing the equation of a line.
12. Graphing a linear function.
13. Finding the intercepts of a function.
14. Finding linear regression using the graphing calculator.
15. Using linear regression to make predictions.
16. Finding the equation of parallel and perpendicular lines.
17. Describing transformation of functions as vertical shifts, horizontal shifts, reflections, stretches, and/or shrinks.
18. Finding the domain of a function.
19. Performing operations with functions.
20. Simplifying composite functions.
21. Finding the inverse of a function.

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|---|---|
| | 22. Graphing a function and its inverse.<br>23. Finding the distance between two points using the distance formula.<br>24. Finding the midpoint between two points using the midpoint formula.<br>25. Writing the equation for a circle in standard form.<br>26. Graphing circles and identifying the domain and range. |

| **Focus Standards Addressed in the Unit** | |
|---|---|
| CC.2.1.HS.F.3 | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| CC.2.1.HS.F.4 | Use units as a way to understand problems and to guide the solution of multi-step problems. |
| CC.2.2.HS.D.7 | Create and graph equations or inequalities to describe numbers or relationships. |
| CC.2.2.HS.C.4 | Interpret the effects transformations have on functions and find the inverses of functions. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.10 | Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. |
| CC.2.2.HS.C.1 | Use the concept and notation of functions to interpret and apply them in terms of their context. |
| CC.2.2.HS.C.6 | Interpret functions in terms of the situation they model. |
| CC.2.4.HS.B.3 | Analyze linear models to make interpretations based on the data. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. Confusing domain and range.<br>2. Evaluating each piece of a piecewise function when given a value to solve for.<br>3. Confusing even and odd functions.<br>4. When finding the domain of a composite rational function, you need just look at your solution. | 1. Domain is the x-values, range is the y-values of the function.<br>2. Find which piece of the function the given value applies to based on the given domain and solve just that piece of the function.<br>3. Even functions have y-axis symmetry, odd functions have origin symmetry.<br>4. When finding the domain of a composite function you must look for restrictions throughout the problem, not just at the solution. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • Basics of Functions and their Graphs<br>• More on Functions and Their Graphs<br>• Linear Functions and Slope<br>• More on Slope<br>• Transformations of Functions<br>• Combinations of Functions; Composite Functions<br>• Inverse Functions<br>• Distance and Midpoint Formulas; Circles | • Determine if a relation is a function<br>• Find the domain and range of a function.<br>• Graph functions.<br>• Determine if a function is even, odd, or neither.<br>• Find the slope of a function.<br>• Find linear regression equation.<br>• Write equations of parallel and perpendicular lines<br>• Describe transformation of functions.<br>• Perform operations with functions.<br>• Determine the domain of composite functions.<br>• Find the inverse of a function.<br>• Find the distance and midpoint between two ordered pairs.<br>• Write the equation for a circle in standard form. | • Relation<br>• Domain<br>• Range<br>• Function<br>• Vertical Line Test<br>• Intercepts<br>• Difference Quotient<br>• Piecewise Function<br>• Increasing<br>• Decreasing<br>• Constant<br>• Relative Maximum<br>• Relative Minimum<br>• Even Function<br>• Odd Function<br>• Step Function |

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|---|---|---|
| | | • Slope<br>• Slope-Intercept Form<br>• Point-Slope Form<br>• Standard Form<br>• Linear Regression<br>• Correlation Coefficient<br>• Parallel Lines<br>• Perpendicular Lines<br>• Average Rate of Change<br>• Vertical Shift<br>• Horizontal Shift<br>• Reflection<br>• Vertical Stretch or Shrink<br>• Horizontal Stretch or Shrink<br>• Transformations<br>• Composite Function<br>• Inverse Function<br>• Horizontal Line Test<br>• Midpoint Formula<br>• Distance Formula<br>• Standard Form of a Circle |

**Assessments**

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**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Meteorology
- Physics
- ADA Regulations
- Microbiology

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Polynomial & Rational Functions

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Polynomial & Rational Functions | 18 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
**Unit 3 – Polynomial & Rational Functions**
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Polynomial & Rational Functions

**Unit Overview**

In this unit, students learn to use graphs of quadratic functions to gain a visual understanding of the algebra that describe our world. Techniques for solving polynomial equations and inequalities as well as rational equations and inequalities are explored to enhance students' ability to manipulate functions algebraically.

| Unit Essential Questions | Key Understandings |
|---|---|
| 1. What are the identifying characteristics of parabolas?<br>2. How are roots of polynomial functions determined?<br>3. How are long division and synthetic division used to divide polynomials?<br>4. How is synthetic division used to the find the zeros of a polynomial functions?<br>5. How are asymptotes found?<br>6. How are polynomial and rational inequalities solved and graphed?<br>7. How are variation problems solved? | 1. Graphing quadratic functions.<br>2. Finding the vertex and axis of symmetry for quadratic functions.<br>3. Determining whether the graph of a quadratic function opens up or down.<br>4. Determining if the vertex of a quadratic function is a maximum or minimum.<br>5. Finding the intercepts of a quadratic function.<br>6. Finding the domain and range of a quadratic function.<br>7. Finding the degree of a polynomial.<br>8. Using the leading coefficient test to determine the end behavior of the graph of a polynomial function.<br>9. Finding the zeros of a polynomial function.<br>10. Using multiplicity to determine if the function crosses the x-axis or touches down and turns around at each zero of the function.<br>11. Showing that there is a real zero between two points on the graph of a polynomial using the intermediate value theorem.<br>12. Finding the number of turning points of a polynomial function.<br>13. Graphing polynomial functions.<br>14. Determining if a polynomial function has even, odd, or neither, symmetry.<br>15. Finding the intercepts of a polynomial function.<br>16. Dividing polynomials using long division and synthetic |

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|---|---|
| | division.<br>17. Using synthetic division and the remainder theorem to evaluate a polynomial function.<br>18. Using synthetic division to find the zeros of a polynomial function.<br>19. Listing all possible rational zeros of a polynomial function.<br>20. Writing a polynomial function given the zeros.<br>21. Using Descartes rule of signs to determine the number of possible real zeros of a polynomial function.<br>22. Finding the domain of rational functions.<br>23. Finding all asymptotes of rational functions.<br>24. Graphing rational functions.<br>25. Solving and graphing polynomial inequalities.<br>26. Solving and graphing rational inequalities.<br>27. Writing variation formulas.<br>28. Finding the constant of variation. |

| **Focus Standards Addressed in the Unit** | |
|---|---|
| CC.2.1.HS.F.7 | Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems. |
| CC.2.2.HS.D.6 | Extend the knowledge of rational functions to rewrite in equivalent forms. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.10 | Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. |
| CC.2.2.HS.C.2 | Graph and analyze functions and use their properties to make connections between the different representations. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. The axis of symmetry is a value.<br>2. Descartes rule of signs is not helpful in determining the zeros of a polynomial function.<br>3. Vertical asymptotes occur at all restrictions on the domain of the rational function. | 1. The axis of symmetry is an equation of the vertical line of symmetry for a quadratic equation.<br>2. Descartes rule of signs can be very helpful in finding the zeros of a polynomial function in that it can narrow down your list of possible real zeros.<br>3. Vertical asymptotes occur at restrictions on the denominator of the rational function after the fraction has been simplified completely. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • Quadratic Functions<br>• Polynomial Functions and Their Graphs<br>• Dividing Polynomials; Remainder and Factor Theorems<br>• Zeros of Polynomial Functions<br>• Rational Functions and Their Graphs<br>• Polynomial and Rational Inequalities<br>• Modeling Using Variation | • Graph quadratic functions.<br>• Graph polynomial functions.<br>• Find the zeros of polynomial functions.<br>• Divide polynomials.<br>• Graph rational functions.<br>• Find the asymptotes of rational functions.<br>• Graph polynomial and rational inequalities.<br>• Write variation equations. | • Standard Form of a Quadratic Function<br>• General Form of a Quadratic Function<br>• Vertex<br>• Opens Up<br>• Opens Down<br>• Maximum<br>• Minimum<br>• Intercepts<br>• Axis of Symmetry<br>• Domain<br>• Range<br>• Smooth<br>• Continuous<br>• Degree of a Polynomial<br>• Leading Coefficient Test<br>• Zeros |

| | | |
|---|---|---|
| | | • Roots<br>• Multiplicity<br>• Intermediate Value Theorem<br>• Turning Points<br>• Even/Odd Symmetry<br>• Long Division<br>• Synthetic Division<br>• Remainder Theorem<br>• Possible Rational Zeros<br>• Complex Zeros<br>• Conjugate Pairs<br>• Descartes Rule of Signs<br>• Vertical Asymptotes<br>• Horizontal Asymptotes<br>• Oblique or Slant Asymptotes<br>• Test Points<br>• Polynomial Inequalities<br>• Rational Inequalities<br>• Constant of Variation<br>• Direct Variation<br>• Inverse Variation<br>• Joint Variation<br>• Combined Variation |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Industrial Design
- Electricity
- Infectious Disease
- Forestry
- Projectile Motion
- Driving
- Sports

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Exponential & Logarithmic Functions

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Exponential & Logarithmic Functions | 19 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
**Unit 4 – Exponential & Logarithmic Functions**
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Exponential & Logarithmic Functions

**Unit Overview**

In this unit students see how exponential and logarithmic functions enable us to predict the future and rediscover the past. Many real-life situations, including population growth, growth of epidemics, radioactive decay, and other changes that involve rapid increase or decrease can be described using exponential functions. The inverse of an exponential function, called the logarithmic function helps students to understand diverse phenomena, including earthquake intensity, human memory, and the pace of life. Students will learn the properties of logarithms and how to apply these properties to simplify expressions and to solve logarithmic equations.

**Unit Essential Questions**

1. How are exponential functions solved and graphed?
2. How are logarithms evaluated?
3. How are logarithms graphed?
4. How are logarithmic expressions expanded or condensed?
5. What is the process for solving exponential and logarithmic equations?
6. How is growth and decay modeled?

**Key Understandings**

1. Graphing exponential functions.
2. Finding equations for horizontal asymptotes of exponential functions.
3. Finding the domain and range of exponential functions.
4. Using transformations to graph exponential functions.
5. Writing logarithms in exponential form and in logarithmic form.
6. Evaluating logarithmic expressions using the properties of logarithms.
7. Evaluating logarithmic expressions using the calculator.
8. Writing natural logarithmic expressions in exponential form and logarithmic form.
9. Evaluating natural logarithmic expressions using the properties of natural logarithms.
10. Evaluating natural logarithms using the calculator.
11. Graphing logarithmic functions.
12. Using transformations to graph logarithmic functions.
13. Finding the domain of logarithmic functions.
14. Using the product, quotient, and power rules to expand and condense logarithmic expressions.
15. Using the change of base formula to convert a logarithmic statement to common logarithms for graphing on the

| | |
|---|---|
| | calculator.<br>16. Solving exponential equations.<br>17. Solving and checking solutions to logarithmic equations.<br>18. Using the exponential growth/decay model to write functions to model data.<br>19. Using the exponential growth/decay model to make predictions about data.<br>20. Using logistic growth model to evaluate functions.<br>21. Modeling data using linear, logarithmic, or exponential functions on the graphing calculator and determining which is the best fit for the data. |

| **Focus Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.2 | Write expressions in equivalent forms to solve problems. |
| CC.2.2.HS.D.8 | Apply inverse operations to solve equations or formulas for a given variable. |
| CC.2.2.HS.D.10 | Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. |
| CC.2.2.HS.C.5 | Construct and compare linear, quadratic, and exponential models to solve problems. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.1 | Interpret the structure of expressions to represent a quantity in terms of its context. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. Logarithmic functions will always be written in logarithmic form.<br>2. Natural logarithmic functions are not related to logarithmic functions.<br>3. Taking the natural logarithm of the numerator of a fraction divided by the natural logarithm of the denominator of a fraction is the same thing as taking the natural logarithm of the quantity of the numerator divided by the denominator.<br>4. LinReg and LnReg are the same thing. | 1. Logarithmic functions can be written in exponential form or in logarithmic form.<br>2. Natural logarithmic functions are logarithmic functions with a base of "e".<br>3. Taking the natural logarithm of the numerator of a fraction divided by the natural logarithm of the denominator of a fraction does not produce the same answer as taking the natural logarithm of the quantity of the numerator divided by the denominator. Watch parenthesis!<br>4. LinReg is linear regression. LnReg is logarithmic regression. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • Exponential Functions<br>• Logarithmic Functions<br>• Properties of Logarithms<br>• Exponential and Logarithmic Equations<br>• Exponential Growth and Decay; Modeling Data | • Graph exponential functions.<br>• Evaluate logarithms.<br>• Graph logarithms.<br>• Expand and condense logarithmic expressions.<br>• Solve exponential and logarithmic equations.<br>• Model exponential growth and decay.<br>• Make predictions about data using exponential growth and decay models.<br>• Evaluate logistic growth models.<br>• Model data using linear, logarithmic, or exponential functions on the graphing calculator. | • Exponential Function<br>• Asymptote<br>• Domain<br>• Range<br>• Irrational Exponential Function<br>• Transformations of Exponential Functions<br>• Logarithmic Functions<br>• Exponential Form of a Logarithmic Function<br>• Logarithmic Form of a Logarithmic Function<br>• Common Logarithmic Function<br>• Natural Logarithmic Function |

| | | |
|---|---|---|
| | | • Exponential Form of a Natural Logarithmic Function<br>• Logarithmic Form of a Natural Logarithmic Function<br>• Transformations of Logarithmic Functions<br>• Product Rule<br>• Quotient Rule<br>• Power Rule<br>• Expand<br>• Condense<br>• Change of Base Formula<br>• Exponential Growth & Decay<br>• Logistic Growth Model<br>• Half-Life<br>• Linear Regression<br>• Logarithmic Regression<br>• Exponential Regression |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Forestry
- Sound Engineering
- Nuclear Science

- Seismology
- Human Memory Model

**Additional Resources:**
Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**
Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Trigonometric Functions

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Trigonometric Functions | 26 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
**Unit 5 – Trigonometric Functions**
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Trigonometric Functions

**Unit Overview**

In this unit, students encounter functions that enable us to model phenomena that occur in cycles. The word trigonometry means "measurement of triangles". Right triangles and the angles and methods for measuring them are explored using the six trigonometric functions. Students will see how to applications of trigonometric functions are used to model phenomena that follow regular, predictable patterns such heartbeats, sleep, seasons, tides, and sound.

**Unit Essential Questions**

1. How are angles converted between degrees and radians?
2. What are the six trigonometric functions and how are they used?
3. What is a reference angle and how are reference angles used to evaluate trigonometric functions?
4. What is meant by even/odd trigonometric functions?
5. How are the six trigonometric functions graphed?
6. How are inverse trigonometric functions evaluated?
7. How do you determine which trigonometric function to use to solve applications of trigonometric functions?

**Key Understandings**

1. Drawing angles in standard position.
2. Converting between degrees and radians.
3. Converting between radians and degrees.
4. Finding the arc length of a circle.
5. Finding the exact value of each trigonometric function for a given angle.
6. Finding approximate values of trigonometric functions using a calculator.
7. Evaluating trigonometric functions using reciprocal, quotient, and Pythagorean identities.
8. Determining which trigonometric function to use to solve applications of trigonometric functions.
9. Evaluating trigonometric functions using the unit circle.
10. Using reference angles to evaluate trigonometric functions exactly.
11. Using even and odd trigonometric properties and periodic properties to determine the solution to trigonometric functions.
12. Determining the amplitude, period, and/or phase shift, and/or vertical shift, and/or reflection for a sine, cosine, secant, and cosecant function, and using this to graph the function.
13. Finding two consecutive vertical asymptotes for tangent and cotangent functions and using these to graph the functions.

| | |
|---|---|
| | 14. Understanding that there are restrictions on the domain of the inverse trigonometric functions.<br>15. Locating the restrictions on the domain of the inverse trigonometric functions.<br>16. Finding the exact value of inverse trigonometric functions<br>17. Finding approximate values of inverse trigonometric functions using the calculator.<br>18. Evaluating composite trigonometric functions exactly.<br>19. Finding bearing.<br>20. Finding the maximum displacement, frequency, and period in simple harmonic motion problems. |

| **Focus Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.1 | Interpret the structure of expressions to represent a quantity in terms of its context. |
| CC.2.2.HS.D.8 | Apply inverse operations to solve equations or formulas for a given variable. |
| CC.2.2.HS.C.7 | Apply radian measure of an angle and the unit circle to analyze the trigonometric functions. |
| CC.2.2.HS.C.8 | Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.1.HS.F.4 | Use units as a way to understand problems and to guide the solution of multi-step problems. |
| CC.2.2.HS.C.2 | Graph and analyze functions and use their properties to make connections between the different representations. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. Finding the reciprocal means to find the inverse of the trigonometric function.<br>2. It doesn't matter when mode I am in when evaluating trigonometric functions on the calculator.<br>3. Squaring a trigonometric function means to square the angle.<br>4. Dividing by zeros equals zero.<br>5. When evaluating inverse trigonometric functions, your reference angle is always your solution. | 1. Inverse is used when you are trying to find the angle of the trigonometric function. Reciprocal is about "flipping over" the trigonometric ratio.<br>2. Pay close attention to what mode you are in when evaluating trigonometric functions.<br>3. Squaring a trigonometric function means the quantity of the trigonometric function of the angle squared.<br>4. You cannot divide by zero. The result is undefined.<br>5. When evaluating inverse trigonometric functions, the reference angle is helpful in obtaining your solution. If you are not in quadrant 1, it is not your solution. The solution is the angle created from the initial side to the terminal side of the reference angle. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • Angles and Radian Measure<br>• Right Triangle Trigonometry<br>• Trigonometric Functions of Any Angle<br>• Trigonometric Functions of Real Numbers; Periodic Functions<br>• Graphs of Sine and | • Convert between radians and degrees and vice versa.<br>• Graph an angle in standard position.<br>• Find the exact value for each trigonometric function given an angle.<br>• Use identities to find the exact value of trigonometric functions.<br>• Use the unit circle to find exact values for each trigonometric function.<br>• Use reference angles to trigonometric functions exactly.<br>• Use even/odd properties to evaluate trigonometric functions exactly.<br>• Graph all six trigonometric functions. | • Radian<br>• Degree<br>• Standard Position of an Angle<br>• Initial Side<br>• Terminal Side<br>• Arc Length<br>• SOHCAHTOA<br>• Sine<br>• Cosine<br>• Tangent<br>• Cosecant |

| | | |
|---|---|---|
| Cosine Functions<br>• Graphs of Other Trigonometric Functions<br>• Inverse Trigonometric Functions<br>• Applications of Trigonometric Functions | • Evaluate inverse trigonometric functions exactly.<br>• Solve applications of trigonometric functions. | • Secant<br>• Cotangent<br>• Opposite<br>• Adjacent<br>• Hypotenuse<br>• Pythagorean Theorem<br>• Special Right Triangles<br>• Exact Value<br>• Reciprocal Identities<br>• Calculator Mode<br>• Quotient Identities<br>• Pythagorean Identities<br>• Inverse Trigonometric Functions<br>• Unit Circle<br>• Reference Angle<br>• All Students Take Calculus (where trig functions are positive)<br>• Even/Odd Properties<br>• Periodic Properties<br>• Amplitude<br>• Period<br>• Phase Shift<br>• Vertical Shift<br>• Reflection<br>• Vertical Asymptotes<br>• Solving a Right Triangle<br>• Bearing<br>• Simple Harmonic Motion<br>• Maximum Displacement<br>• Frequency |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

## Suggested Strategies to Support Design of Coherent Instruction

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Respiration
- Extreme Sports
- Meteorology
- Building & Design
- Engineering

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Analytic Trigonometry

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Analytic Trigonometry | 17 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
**Unit 6 – Analytic Trigonometry**
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Analytic Trigonometry

**Unit Overview**

In this unit, students learn how to verify a trigonometric identity by showing that one side of the identity can be simplified so that it is identical to the other side. Proving a trigonometric relationship requires students to be creative in their approach to problem solving. By learning to establish these relationships, students will become a better, more confident problem solver. In this unit students will also learn how to apply the trigonometric identities to solving trigonometric equations.

**Unit Essential Questions**

1. What does it mean to verify a trigonometric identity?
2. How do you verify a trigonometric identity?
3. How are the sum and difference formulas applied?
4. How are the double angle, half-angle, and power reducing formulas applied?
5. How are the product-to-sum and sum-to-product formulas applied?
6. How are trigonometric equations solved?

**Key Understandings**

1. Verifying trigonometric identities means to make both sides of the identity equal.
2. Verifying trigonometric identities involves working with one side of the identity independent of the other side until the two sides are equal.
3. Manipulating the Pythagorean identities for use in verifying trigonometric identities.
4. Using sum and difference formulas to evaluate trigonometric functions exactly.
5. Using sum and difference formulas to verify trigonometric identities.
6. Using double angle, half-angle, and power reducing formulas to evaluate trigonometric functions exactly.
7. Using double angle, half-angle, and power reducing formulas to verify trigonometric identities.
8. Using product-to-sum and sum-to-product formulas to evaluate trigonometric functions exactly.
9. Using product-to-sum and sum-to-product formulas to verify trigonometric identities.
10. Solving trigonometric equations over a given domain.
11. Finding approximate solutions to trigonometric equations over a given domain using the graphing calculator.

| Focus Standards Addressed in the Unit | |
|---|---|
| CC.2.3.HS.A.7 | Apply trigonometric ratios to solve problems involving right triangles. |
| CC.2.2.HS.C.8 | Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs. |
| CC.2.2.HS.C.9 | Prove the Pythagorean identity and use it to calculate trigonometric ratios. |

| Important Standards Addressed in the Unit | |
|---|---|
| CC.2.2.HS.D.2 | Write expressions in equivalent forms to solve problems. |

**Misconceptions**

1. Verifying a trigonometric identity is impossible. I can't do it! I just stare at it.
2. When verifying a trigonometric identity sometimes a negative disappears.

**Proper Conceptions**

1. Start on the more complicated looking side of the identity. Apply a fundamental property. Try changing everything to sine and/or cosine. Try, try, again! Just staring at the identity is not going to get you anywhere.
2. When evaluating a trigonometric identity don't forget about using even/odd trigonometric properties which deal with negatives.

**Concepts**

- Verifying Trigonometric Identities
- Sum and Difference Formulas
- Double-Angle, Power-Reducing, and Half-Angle Formulas
- Product-to-Sum and Sum-to-Product Formulas
- Trigonometric Equations

**Competencies**

- Verify trigonometric identities.
- Apply sum and difference formulas.
- Apply double angle, half-angle, and power reducing formulas.
- Apply product-to-sum and sum-to-product formulas.
- Solve trigonometric equations.

**Vocabulary**

- Verify Trigonometric Identities
- Manipulating Trigonometric Identities
- Sum and Difference Formulas
- Double Angle Formulas
- Half-Angle Formulas
- Power Reducing Formulas
- Product-to-Sum Formula
- Sum-to-Product Formulas
- Solving Trigonometric Equations
- All Students Take Calculus (where trig functions are positive)
- Check Solutions

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**
*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
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3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Standing Waves
- Projectile Motion
- Friction

**Additional Resources:**
Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**
Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Additional Topics in Trigonometry

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Additional Topics in Trigonometry | 23 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
**Unit 7 – Additional Topics in Trigonometry**
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Additional Topics in Trigonometry

**Unit Overview**

In this unit, students move beyond right triangle trigonometry to solving oblique triangles using the Law of Sines or Law of Cosines. Students will learn the different cases for solving oblique triangle and understand which law applies to solving the triangle. Finding the area of oblique triangles is also a major focus of the unit. Another big concept of this unit is the polar coordinate system. Students learn how to graph polar equations and complex numbers including writing them in terms of trigonometric functions. In the final component of this unit, students are introduced to the world of vectors, which literally surround our every move.

**Unit Essential Questions**

1. What kinds of triangles can be solved using the Law of Sines?
2. What kinds of triangles can be solved using the Law of Cosines?
3. How are points plotted in the polar coordinate system?
4. How are polar equations graphed?
5. How do you convert between polar and rectangular forms of complex numbers?
6. How are vectors represented?
7. What are orthogonal vectors?

**Key Understandings**

1. Solving oblique triangles with the Law of Sines when given AAS, ASA, and SSA.
2. Checking for one, two, or no triangles when given SSA.
3. Finding the area of an oblique triangle when given SAS.
4. Solving oblique triangles with the Law of Cosines when given SAS or SSS.
5. Finding the area of an oblique triangle when given SSS.
6. Plotting points in the polar coordinate system.
7. Converting from polar coordinates to rectangular coordinates.
8. Converting from rectangular coordinates to polar coordinates.
9. Converting rectangular equations to polar equations.
10. Converting polar equations to rectangular equations.
11. Graphing polar equations.
12. Identifying symmetry of polar equations.
13. Writing complex numbers in polar form.
14. Writing complex numbers in rectangular form.
15. Finding the product of two complex numbers.
16. Finding the quotient of tow complex numbers.
17. Using Demoivre's Theorem to reduce the power of complex numbers.

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|---|---|
| | 18. Using Demoivre's Theorem to find complex roots.<br>19. Drawing vectors.<br>20. Finding a position vector.<br>21. Finding the magnitude of a vector.<br>22. Proving two vectors are equal.<br>23. Finding a unit vector.<br>24. Writing a vector in terms of its magnitude and direction angle.<br>25. Finding the dot product.<br>26. Finding the angle between two vectors.<br>27. Determining if two angles are orthogonal.<br>28. Decomposing a vector into two vectors such that one is parallel and the other is orthogonal to a given vector. |

| **Focus Standards Addressed in the Unit** | |
|---|---|
| CC.2.1.HS.F.6 | Extend the knowledge of arithmetic operations and apply to complex numbers. |
| CC.2.1.HS.F.7 | Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems. |
| CC.2.2.HS.D.7 | Create and graph equations or inequalities to describe numbers or relationships. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.2 | Write expressions in equivalent forms to solve problems. |
| CC.2.3.HS.A.14 | Apply geometric concepts to model and solve real world problems. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. Polar symmetry and polar axis symmetry are the same type of symmetry.<br>2. All forms of oblique triangle solutions have been studied in prior course work.<br>3. Complex numbers cannot be graphed. | 1. Symmetry with respect to the pole is origin symmetry which is not the same thing as polar axis symmetry which is x-axis symmetry.<br>2. The SSA triangle configuration requires understanding of circular trigonometry.<br>3. There is a complex plane for graphing complex numbers. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • The Law of Sines<br>• The Law of Cosines<br>• Polar Coordinates<br>• Graphs of Polar Equations<br>• Complex Numbers in Polar Form; Demoivre's Theorem<br>• Vectors<br>• The Dot Product | • Solve oblique triangles with the Law of Sines.<br>• Solve oblique triangles with the Law of Cosines.<br>• Find the area of oblique triangles.<br>• Plot points in the polar coordinate system.<br>• Convert points between polar and rectangular forms.<br>• Convert equations between polar and rectangular forms.<br>• Graph polar equations.<br>• Convert between polar and rectangular form of complex numbers.<br>• Represent vectors in multiple forms.<br>• Find the dot product. | • Law of Sines<br>• AAS<br>• ASA<br>• SSA<br>• Area of Oblique Triangles<br>• Law of Cosines<br>• SSS<br>• SAS<br>• Heron's Formula<br>• Polar Coordinates<br>• Pole<br>• Polar Axis<br>• Line<br>• Circle<br>• Lemniscate<br>• Rose<br>• Limacon<br>• Limacon with Inner Loop<br>• Cardiod |

| | | |
|---|---|---|
| | | • Limacon with Dimple<br>• Limacon without Dimple<br>• Polar Symmetry<br>• Polar Axis Symmetry<br>• Symmetry with respect to the Line<br>• Complex Plane<br>• Product of Complex Numbers<br>• Quotient of Complex Numbers<br>• Demoivre's Theorem for Powers of Complex Numbers<br>• Demoivre's Theorem for Complex Roots<br>• Vectors<br>• Initial Pointr<br>• Terminal Point<br>• Position Vector<br>• Magnitude of a Vector<br>• Equal Vectors<br>• Unit Vector<br>• Writing a Vector in Terms of its Magnitude and Direction<br>• Dot Product<br>• Angle Between Two Vectors<br>• Orthogonal Vectors<br>• Parallel Vectors |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Navigation
- Surveying
- Electrical Engineering

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Systems of Equations & Inequalities

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Systems of Equations & Inequalities | 16 Days |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
**Unit 8 – Systems of Equations & Inequalities**
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Systems of Equations & Inequalities

**Unit Overview**

In this unit, students expand their knowledge of solving systems of equations and inequalities to systems that contain more than two variables to systems that include quadratics. Students will also learn a technique used in calculus to find a function if its rate of change is known. The technique involves expressing a given function in terms of simpler functions – partial fraction decomposition. The unit concludes with a study of linear programming, a method for solving problems in which a particular quantity that must be maximized or minimized is limited by other factors. Used in management science, it helps businesses allocate resources to manufacture products in a way that will maximize profit.

**Unit Essential Questions**

1. How are systems of linear equations in two variables solved?
2. How are systems of linear equations in three variables solved?
3. How are rational expressions decomposed into partial fractions?
4. How are systems of nonlinear equations in two variables solved?
5. How are systems of inequalities solved?
6. How is linear programming used to solve problems?

**Key Understandings**

1. Finding the solution to a system of linear equations in two variables.
2. Eliminating a variable method for solving a system of linear equations in two variables.
3. Graphing method for solving a system of linear equations in two variables.
4. Substitution method for solving a system of linear equations in two variables.
5. Determining when a system of linear equations is inconsistent.
6. Determining when a system of linear equations is dependent.
7. Finding the break-even point for a system of linear equations.
8. Finding the solution to a system of linear equations in three variables.
9. Eliminating variables method for solving a system of linear equations in three variables.
10. Graphing method for solving a system of linear equations in three variables.
11. Substitution method for solving a system of linear equations in three variables.

| | |
|---|---|
| | 12. Decomposing a rational expression into partial fractions.<br>13. Finding the solution to a system of nonlinear equations in two variables.<br>14. Elimination all occurrences of one of the variables by the substitution or addition methods.<br>15. Graphing a system of inequalities.<br>16. Maximizing and minimizing an objective function.<br>17. Graphing the region determined by given constraints.<br>18. Solve linear programming problems. |

| **Focus Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.7 | Create and graph equations or inequalities to describe numbers or relationships. |
| CC.2.2.HS.D.10 | Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. |
| CC.2.3.HS.A.14 | Apply geometric concepts to model and solve real world problems. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.C.6 | Interpret functions in terms of the situations they model. |
| CC.2.4.HS.B.3 | Analyze linear models to make interpretations based on the data. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. Linear systems are limited to two variables.<br>2. Systems of inequalities are limited to two inequalities.<br>3. Linear programming problems have infinite solutions without a "best case" scenario. | 1. Although many linear systems contain two variables, many applications require systems beyond two variables.<br>2. Systems of inequalities in manufacturing situations often have more than two linear inequalities as part of the system.<br>3. It is true that solution regions have an infinite possible number of solution points. However, the solution point that will maximize or minimize the objective function can be found at one of the vertices in the feasible region. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • Systems of Linear Equations in Two Variables<br>• Systems of Linear Equations in Three Variables<br>• Partial Fractions<br>• Systems of Nonlinear Equations in Two Variables<br>• Systems of Inequalities<br>• Linear Programming | • Solve systems of linear equations in two variables.<br>• Solve systems of linear equations in three variables.<br>• Decompose rational expressions into partial fractions.<br>• Solve systems of nonlinear equations in two variables.<br>• Solve systems of inequalities.<br>• Solve problems using linear programming. | • System of Linear Equations<br>• Solution to a System of Linear Equations<br>• Substitution Method<br>• Elimination Method<br>• Graphing Method<br>• Inconsistent System<br>• Identity<br>• Dependent<br>• Revenue Function<br>• Cost Function<br>• Break-Even Point<br>• Profit Function<br>• Solution Set<br>• Reducing the System<br>• Ordered Triple<br>• Partial Fraction Decomposition<br>• Partial Fraction<br>• Linear Factor<br>• Constant |

| | | |
|---|---|---|
| | | • Quadratic Factor<br>• System of Nonlinear Equations<br>• Substitution Method<br>• Elimination Method<br>• Graphing Method<br>• System of Inequalities<br>• Test Point<br>• Linear Programming<br>• Objective Function<br>• Constraints |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Traffic Flow
- Fuel Mixture
- Global Positioning System
- Waste Water Infrastructure
- Nutrition
- Management Science

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**
Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Matrices & Determinants

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Matrices & Determinants | 17 Days *if time permits |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
**Unit 9 – Matrices & Determinants**
Unit 10 – Conic Sections & Analytic Geometry
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Matrices & Determinants

**Unit Overview**

In this unit, students learn to represent information, perform matrix operations, use inverses, and solve various systems using matrices. Matrix algebra has various applications from using your smartphone to read your e-mail, to using the internet to browse through art museums and photography exhibits.

**Unit Essential Questions**

1. How are systems of equations solved using matrices?
2. How is Gaussian elimination used to find the complete solution to systems of equations?
3. How are operation with matrices performed?
4. How is the multiplicative inverse of matrices found?
5. How are matrix equations solved?
6. How are determinants determined?
7. How is Cramer's Rule used to solve a system of equations?

**Key Understandings**

1. Solving linear systems using Gaussian elimination.
2. Writing a matrix in row-echelon form.
3. Solving linear systems using Gauss-Jordan elimination.
4. Writing a matrix in reduced row-echelon form.
5. Determining if a system is inconsistent using Gaussian elimination.
6. Determining if a system is dependent using Gaussian elimination.
7. Determining if two matrices are equal.
8. Performing matrix addition and subtraction.
9. Performing scalar multiplication.
10. Performing matrix multiplication.
11. Finding the multiplicative inverse.
12. Representing linear systems by matrix equations.
13. Using Cramer's Rule for solving systems of linear equations.
14. Evaluating an nth-order determinant.

**Focus Standards Addressed in the Unit**

| | |
|---|---|
| CC.2.1.HS.F.3 | Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays. |
| CC.2.2.HS.D.8 | Apply inverse operations to solve equations or formulas for a given variable. |

| Important Standards Addressed in the Unit | |
|---|---|
| CC.2.2.HS.D.1 | Interpret the structure of expressions to represent a quantity in terms of its context. |
| CC.2.2.HS.C.6 | Interpret functions in terms of the situations they model. |
| CC.2.3.HS.A.14 | Apply geometric concepts to model and solve real world problems. |

| Misconceptions | Proper Conceptions |
|---|---|
| 1. Matrix multiplication is commutative – that is, the order you multiply matrices doesn't matter.<br>2. Cramer's Rule can determine if a system has no solution or infinitely many solutions. | 1. Matrix multiplication is not commutative.<br>2. Cramer's Rule cannot be used to determine solution sets with inconsistent or dependent systems. |

**Concepts**

- Matrix Solutions to Linear Systems
- Inconsistent and Dependent Systems and Their Applications
- Matrix Operations and Their Applications
- Multiplicative Inverses of Matrices and Matrix Equations
- Determinants and Cramer's Rule

**Competencies**

- Solve linear systems using matrices.
- Determine if a system is inconsistent or dependent using matrices.
- Perform operations with matrices.
- Solve applications with matrices.
- Find the multiplicative inverse for matrices.
- Use Cramer's Rule to solve systems of linear equations.
- Evaluate nth-order determinants.

**Vocabulary**

- Matrix
- Matrices
- Augmented Matrix
- Main Diagonal
- Row-Echelon Form
- Row Operations
- Row Equivalent
- Gaussian Elimination
- Gauss-Jordan Elimination
- Reduced Row-Echelon Form
- Inconsistent System
- Dependent System
- Nonsquare System
- Order m x n
- Square Matrix
- Equal Matrices
- Zero Matrix
- Additive Identity
- Additive Inverse
- Scalar
- Scalar Multiple
- Properties of Matrix Multiplication
- Pixels
- Multiplicative Identity Matrix of Order n
- Multiplicative Inverse
- Invertible or NonSingular
- Singular
- Coefficient Matrix
- Column Matrices
- Constant Matrix
- Coding Matrix
- Coded Matrix
- Determinant
- Second-Order Determinant
- Nth – Order Determinant
- Cramer's Rule
- Minor
- Cofactor

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Health and Wellness
- Photography
- Beam Deflection
- Networking
- Data Encrypton
- Encoding/Decoding

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Conic Sections & Analytic Geometry

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Conic Sections & Analytic Geometry | 19 Days *if time permits |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
**Unit 10 – Conic Sections & Analytic Geometry**
Unit 11 – Sequences, Induction, & Probability

**Unit Title**

Conic Sections & Analytic Geometry

**Unit Overview**

In this unit, students use the rectangular coordinate system to study the conic sections: Parabolas, Ellipses, Hyperbolas, Circles, Lines, and Point, and the mathematics behind their surprising applications. Students will looks at ways of describing curves that reveal the where and when of motion including conic section in polar coordinates.

**Unit Essential Questions**

1. How is an ellipse graphed?
2. How is a hyperbola graphed?
3. How is a parabola graphed?
4. How is a rotated system graphed?
5. How are plane curves described by parametric equations graphed?
6. How are polar equations of conics graphed?

**Key Understandings**

1. Identifying the characteristics of an ellipse.
2. Writing the equation of ellipses in standard form.
3. Graphing ellipses.
4. Identifying the characteristics of a hyperbola.
5. Writing the equation of hyperbolas in standard form.
6. Graphing hyperbolas.
7. Identifying the characteristics of a parabola.
8. Writing the equation of a parabola in standard form.
9. Graphing parabolas.
10. Determining what conic section is being represented by equations.
11. Rotating axes.
12. Graphing rotated systems.
13. Graphing plane curves described by parametric equations using point plotting.
14. Graphing plane curves by eliminating the parameter and graphing the resulting rectangular equation.
15. Identifying the conic section represented by a polar equation.
16. Graphing polar equations of conics.

**Focus Standards Addressed in the Unit**

| | |
|---|---|
| CC.2.2.HS.D.2 | Write expressions in equivalent forms to solve problems. |

| | |
|---|---|
| CC.2.2.HS.D.5 | Use polynomial identities to solve problems. |
| CC.2.2.HS.C.2 | Graph and analyze functions and use their properties to make connections between the different representations. |
| CC.2.3.HS.A.10 | Translate between the geometric description and the equation for a conic section. |

| **Important Standards Addressed in the Unit** | |
|---|---|
| CC.2.2.HS.D.10 | Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. |
| CC.2.2.HS.C.6 | Interpret functions in terms of the situations they model. |

| **Misconceptions** | **Proper Conceptions** |
|---|---|
| 1. Circles and parabolas are unique entities.<br>2. Locating an ellipse's foci is the same process as locating a hyperbola's foci. | 1. These two commonly studied items are both Conic Sections – they are formed by passing a plane through a cone. And, there are more conic sections other than the circle and parabola.<br>2. There is a sign difference between location an ellipse's foci and a hyperbola's foci. |

| **Concepts** | **Competencies** | **Vocabulary** |
|---|---|---|
| • The Ellipse<br>• The Hyperbola<br>• The Parabola<br>• Rotation of Axes<br>• Parametric Equations<br>• Conic Sections in Polar Coordinates | • Graph ellipses.<br>• Graph hyperbolas.<br>• Graph parabolas.<br>• Graph a rotated conic section.<br>• Graph plane curves described by parametric equations.<br>• Graph polar equations of conics. | • Ellipse<br>• Foci<br>• Focus<br>• Center<br>• Vertex<br>• Vertices<br>• Major Axis<br>• Minor Axis<br>• Standard From of the Equation of an Ellipse<br>• Hyperbola<br>• Transverse Axis<br>• Standard From of the Equation of a Hyperbola<br>• Asymptotes<br>• Conjugate Axis<br>• Branches of the Hyperbola<br>• Parabola<br>• Directrix<br>• Focus<br>• Axis of Symmetry<br>• Standard Form of the Equation of a Parabola<br>• Latus Rectum<br>• Degenerate Conic Sections<br>• General Second-Degree Equation<br>• Rotation of Axes<br>• Conic Sections<br>• Parameter<br>• Parametric Equations<br>• Plane Curve<br>• Orientation<br>• Eliminating the |

| | | |
|---|---|---|
| | | Parameter<br>• Cycloid<br>• Eccentricity |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Sound Engineering
- Nuclear Engineering
- Radio Telescopes
- Satellite Orbit
- Medicine

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols

# Academic Trigonometry & Advanced Math / 11 -12

## Sequences, Induction, & Probability

| Subject | Grade | Unit | Suggested Timeline |
|---|---|---|---|
| Mathematics | 11/12 | Sequences, Induction, & Probability | 21 Days *if time permits |

**Grade Level Summary**

Unit P - Prerequisites: Fundamental Concepts of Algebra
Unit 1 – Equations & Inequalities
Unit 2 – Functions & Graphs
Unit 3 – Polynomial & Rational Functions
Unit 4 – Exponential & Logarithmic Functions
Unit 5 – Trigonometric Functions
Unit 6 – Analytic Trigonometry
Unit 7 – Additional Topics in Trigonometry
Unit 8 – Systems of Equations & Inequalities
Unit 9 – Matrices & Determinants
Unit 10 – Conic Sections & Analytic Geometry
**Unit 11 – Sequences, Induction, & Probability**

**Unit Title**

Sequences, Inductions, & Probability

**Unit Overview**

In this unit, students will discuss the characteristics of arithmetic and geometric sequences and series. An investigation into the principle of mathematical induction follows as students learn to write proofs of statements. Counting principles for combinations and permutations is a final progression leading into investigations for simple and combined probabilities.

**Unit Essential Questions**

1. How are sequences represented?
2. What is an arithmetic sequence?
3. What is a geometric sequence?
4. How is mathematical induction used?
5. How is the binomial theorem applied?
6. What applications require the fundamental counting principle?
7. How are permutations and combinations performed?
8. How is probability of events determined?

**Key Understandings**

1. Writing terms of a sequence.
2. Evaluating factorial expression.
3. Evaluating summation expressions.
4. Expressing sums using summation notation.
5. Writing terms of an arithmetic sequence.
6. Writing formulas of arithmetic sequences.
7. Finding the sum of terms of an arithmetic sequence.
8. Writing terms of a geometric sequence.
9. Writing formulas of geometric sequences.
10. Finding the sum of terms of a geometric sequence.
11. Using mathematical induction to prove that statements are true.
12. Using the binomial theorem to expand binomials.
13. Finding the indicated term of a binomial expression.
14. Determining if a permutation or a combination is used to solve application problems.
15. Evaluate expressions using permutations.
16. Evaluate expressions using combinations.
17. Using the fundamental counting principle to solve application problems.
18. Solving probability application problems.

| Focus Standards Addressed in the Unit | |
|---|---|
| CC.2.4.HS.B.4 | Recognize and evaluate random processes underlying statistical experiments. |
| CC.2.4.HS.B.6 | Use the concepts of independence and conditional probability to interpret data. |
| CC.2.4.HS.B.7 | Apply the rules of probability to compute probabilities of compound events in a uniform probability model. |

| Important Standards Addressed in the Unit | |
|---|---|
| CC.2.2.HS.D.2 | Write expressions in equivalent forms to solve problems. |
| CC.2.4.HS.B.5 | Make inferences and justify conclusions based on sample surveys, experiments, and observational studies. |

**Misconceptions**

1. Sequences have a common difference or common ratio.
2. Probabilities are the same as odds.
3. The formula for permutations must be used to solve permutation problem.

**Proper Conceptions**

1. Arithmetic and geometric are only two common types of sequences.
2. Probability is the number of favorable outcomes compared to the total number of possible outcomes. Odds are the number of favorable outcomes compared to the number of non-favorable outcomes.
3. All permutation problems are also fundamental counting problems, they can be solved using the fundamental counting principle or the permutations formula.

**Concepts**

- Sequences and Summation Notation
- Arithmetic Sequences
- Geometric Sequences and Series
- Mathematical Induction
- The Binomial Theorem
- Counting Principles, Permutations, and Combinations
- Probability

**Competencies**

- Express sequences using summation notation.
- Solve factorials.
- Recognize, write, and solve arithmetic sequences.
- Recognize, write, and solve geometric sequences.
- Use mathematical induction in proofs.
- Expand a binomial raised to a power and find a particular term of a binomial expansion.
- Solve counting problems using the fundamental counting principle.
- Determine whether the permutation or combination formula is needed to solve counting problems and then solve the problem using the appropriate formula.
- Determine the probability of an event occurring.

**Vocabulary**

- Fibonacci sequence
- General Term
- Infinite Sequence
- Finite Sequence
- Recursion Formulas
- Factorial Notation
- Summation Notation
- Index of Summation
- Upper Limit Summation
- Lower Limit Summation
- Expanding the Summation Notation
- Arithmetic Sequence
- Common Difference
- Nth Partial Sum
- Geometric Sequence
- Common Ratio
- Annuity
- Geometric Series
- Infinite Geometric Series
- Multiplier Effect
- Mathematical Induction
- Binomial Coefficient
- Binomial Theorem
- Pascal's Triangle
- Fundamental Counting Principle
- Permutation
- Combination

| | | |
|---|---|---|
| | | • Empirical Probability<br>• Theoretical Probability<br>• Mutually Exclusive<br>• Independent Events<br>• Dependent Events |

**Assessments**

**Homework** – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

**Class Notebook Checks** – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

**Quizzes** – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

**Unit Test** – Each unit will include a summative written test.

**Unit Project** – Typically, each unit will include a project which infuses calculus concepts with prior knowledge and extends understanding through the integration of technology.

**Suggested Strategies to Support Design of Coherent Instruction**

*Charlotte Danielson's Framework for Teaching: Domain 3 Instruction*

3a – Student assignment sheets communicate expectations for learning.
3b. – Using questioning and discussion techniques – connects to implicit differentiation.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

**Differentiation:**

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook formats

**Interdisciplinary Connections:**

- Medicine
- License and Phone Numbers – Combinatorics
- Electricity
- Business
- Gambling

**Additional Resources:**

Kahn Academy
Textbook Ancillary Materials
Chapter Test Prep CD's
Student notes from prior coursework

**Created By:**

Kathleen Nichols