Transitions to College Mathematics / 11-12 /
Basic Algebraic Operations

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Basic Algebraic Operations	30 days

Grade Level Summary

This course is designed to bridge the gap between Algebra II, Geometry, and collegiate courses in mathematics. Emphasis will be placed on linear, polynomial, rational, trigonometric, exponential, and logarithmic functions, matrices, systems of equations and inequalities, and other algebraic and geometric concepts. Students may use graphing calculators and computer software for various mathematical applications. Both topics and depth of study aim to be consistent with the expectations of a traditional College Algebra course required for many non-math dependent collegiate majors or many associate degree programs.

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry-
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Basic Algebraic Operations

Unit Overview

"Algebra is 'generalized arithmetic.' In arithmetic we add, subtract, multiply, and divide specific numbers. In algebra we use all that we know about arithmetic, but in addition, we work with symbols that represent on o r more numbers. IN this [unit] we review some important basic algebraic operations usually studied in earlier courses." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

1.1 How do you add, subtract, multiply, and divide real numbers?
What are the properties of real numbers?

1.2 How do you simplify exponents?
What are the properties of radicals?
How do you convert between rational exponent and radical forms?

1.3 How do you add, subtract, multiply, and divide polynomials?
How do you factor polynomials?

1.4 How do you simplify rational expressions?
How do you add, subtract, multiply, and divide rational expressions?

Key Understandings

1. Operating over the real number system.
2. Using and identifying the properties of algebra
3. Simplifying exponents
4. Simplifying radicals
5. Operating with polynomials
6. Factoring
7. Operating with rational expressions

Focus Standards Addressed in the Unit	
CC.2.1.HS.F.1	Apply and extend the properties of exponents to solve problems with rational exponents.
CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.

Important Standards Addressed in the Unit	

Misconceptions	Proper Conceptions
1. The square of a sum/difference is the sum/difference of its squares: $(a+b)^2 = a^2 + b^2$	1. The square of a sum/difference is found by FOILing a binomial: $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$
2. Negative exponents yield negative numbers: $x^{-1} = -x$	2. Negative exponents yield reciprocals: $x^{-1} = 1/x$
3. The square of a negative number and the negative of a square number are the same: $(-x)^2 = -x^2$	3. The square of a negative number is a positive number and the negative of a square number is negative: $(-x)^2 = x^2$ AND $-(x^2) = -x^2$
4. When simplifying rational expressions, individual terms cancel.	4. When simplifying rational expressions, common factors of the numerator and denominator reduce.
5. When *multiplying and dividing* rational expressions, common denominators are necessary.	5. When *adding or subtracting* rational expressions, common denominators are necessary.

Concepts	Competencies	Vocabulary
1.1 Algebra and Real Numbers	1.1.a. Students should be able to identify and classify the subsets of real numbers. 1.1.b. Students should be able to operate (add, subtract, multiply and divide) over the real numbers. 1.1.c. Students should be able to identify and use the properties of real numbers.	Real numbers Set of real numbers Set of integers element subset empty/null set natural numbers
1.2 Exponents and Radicals	1.2.a. Students should be able to identify and use the properties of integer exponents. 1.2.b. Students should be able to convert between scientific notation and standard decimal form. 1.2.c. Students should be able to convert between rational exponent and radical forms. 1.2.d. Students should be able to simplify and operate with rational exponents and radicals.	integers rational numbers irrational numbers equal sets real number line coordinate origin Closure property Associative property
1.3 Polynomials: Basic Operations and Factoring	1.3.a. Students should be able to identify and classify polynomials. 1.3.b. Students should be able to operate (add, subtract, multiply) with polynomials. 1.3.c. Students should be able to factor polynomials completely.	Commutative property Identity property Inverse property Distributive property Exponent Base Scientific notation
1.4 Rational Expressions: Basic Operations	1.4.a. Students should be able to simply rational expressions completely. 1.4.b. Students should be able to operate (add, subtract, multiply, and divide) with rational expressions.	Square root Cube root Radical Index Principal nth root Simplified form Conjugate

		Algebraic expression Polynomial Degree of term Degree of polynomial Monomial Binomial Trinomial Coefficient Like terms Factor Prime Composite Least Common Denominator Compound Fraction reciprocal

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3c – Instructional materials and unit project activities engage students in learning.
3d – Daily informal assessments of student understanding is provided through skeletal classroom notes, homework and continued student/teacher interaction.
3e – Adjustment to pacing and additional examples and/or practice is used as feedback merits.

Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook and online formats

Interdisciplinary Connections:

Economics
Physics
Chemistry
Geometry
Construction/Architecture

Additional Resources:

Kahn Academy

Created By:

Rebecca Myers

Transitions to College Mathematics / 11-12 / Equations and Inequalities

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Equations and Inequalities	25 days

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Equations and Inequalities

Unit Overview

"Solving equations and inequalities is one of the most important skills in algebra because it can be applied to solving a boundless supply of real-world problems. In this chapter, we will begin with a look at techniques for solving linear equations and inequalities. After a study of complex numbers, we'll return to equations, learning how to solve a variety of nonlinear equations. For each type of equation and inequality we solve, we will look at some real-world problems that can be solved using those solution techniques." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

2.1 How do you solve linear equations?
How can you use linear equations to solve real world problems?

2.2 How do you convert between inequality and interval notations?
How do you solve linear inequalities?

2.3 How do you solve absolute value equations and inequalities?

2.4 How do you add, subtract, multiply, and divide complex numbers?
How do you solve complex equations?

2.5 What are the methods for solving quadratic equations?
How can you use quadratic equations to solve word problems?

2.6 How do you solve radical equations?
How do you solve quadratic-type equations?

Key Understandings

1. Solving equations: linear, absolute value, complex, and quadratic
2. Solving inequalities
3. Operating over the complex number system
4. Completing the square

Focus Standards Addressed in the Unit

CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Important Standards Addressed in the Unit

Misconceptions	**Proper Conceptions**
1. When multiplying to eliminate fractions in an equation, you only need to multiply the fractions and not the whole numbers by the LCD: $\frac{1}{x}\bullet x+1=\frac{2}{x}\bullet x \rightarrow 1+1=2$	1. When multiplying to eliminate fractions in an equation, you must distribute the LCD completely through both sides: $\frac{1}{x}\bullet x+1\bullet x=\frac{2}{x}\bullet x \rightarrow 1+x=2$
2. When multiplying to eliminate fractions in an equation, you need to multiply the numerator and the denominator by the LCD: $\frac{3}{x}\bullet\frac{2x}{2x}=\frac{5}{2}\bullet\frac{2x}{2x} \rightarrow \frac{6x}{2x^2}=\frac{10x}{4x}$	2. When multiplying to eliminate fractions in an equation, you only multiply the numerator by the LCD: $\frac{3}{x}\bullet 2x=\frac{5}{2}\bullet 2x \rightarrow 6=5x$
3. Multiplying or dividing by a negative number does not change the inequality symbol.	3. Multiplying or dividing by a negative number changes the inequality symbol.
4. The absolute value of a difference is simplified by first making all signs positive, then combining like terms: $\lvert x-2x\rvert=\lvert x+2x\rvert=3x$	4. The absolute value of a difference is simplified by first combining like terms, then making the result positive: $\lvert x-2x\rvert=\lvert -x\rvert=x$
5. The product of two square roots of negative numbers is the square root of a positive number: $\sqrt{-2}\bullet\sqrt{-2}=\sqrt{4}=2$	5. The product of two square roots of negative numbers is the square root of a negative number: $\sqrt{-2}\bullet\sqrt{-2}=i\sqrt{2}\bullet i\sqrt{2}=i^2\sqrt{4}=-2$
6. When squaring both sides of an equation, the square of a sum/difference is the sum/difference of its squares: $(a+b)^2=a^2+b^2$	6. When squaring both sides of an equation, the same rule for squaring binomials applies, the square of a sum/difference is found by FOILing a binomial: $(a+b)^2=(a+b)(a+b)=a^2+2ab+b^2$
7. In a quadratic equation, if $a\neq 1$, you do not need to divide both sides by a before completing the square.	7. In a quadratic equation, if $a\neq 1$, you must divide both sides by a before completing the square.

Concepts	**Competencies**	**Vocabulary**
2.1 Linear Equations Applications	2.1.a. Students should be able to solve linear equations. 2.1.b. Students should be able to apply concepts of linear equations to solve real-world problems.	Algebraic equation Domain Solution set Solve
2.2 Linear Inequalities	2.2.a. Students should be able to convert between inequality and interval notations. 2.2.b. Students should be able to solve linear inequalities 2.2.c. Students should be able to apply concepts of linear inequalities to solve real-world problems.	Identity Conditional equation Equivalent equations Standard form of a linear equation Inequality Open interval
2.3 Absolute Value in Equations and inequalities	2.3.a. Students should be able to related absolute value and distance. 2.3.b. Students should be able to solve absolute value and inequality problems. 2.3.c. Students should be able to use absolute value to solve radical inequalities.	Closed interval Union Intersection Absolute value Distance on a number line
2.4 Complex Numbers	2.4.a. Students should be able to classify the components of complex numbers. 2.4.b. Students should be able to operate (add, subtract,	Imaginary unit Complex number Standard form of a complex number Pure imaginary number

	multiply, divide) over the complex numbers. 2.4.c. Students should be able to relate complex numbers to radicals 2.4.d. Students should be able to solve equations involving complex numbers.	Conjugate of a complex number Principal square root of a negative real number Quadratic equation Standard form of quadratic equation
2.5 Quadratic Equations and Applications	2.5.a. Students should be able to solve quadratic equations by factoring, the square root property, completing the square, and the quadratic formula. 2.5.b. Students should be able to apply concepts of quadratic equations to solve real-world problems.	Real root Imaginary root Zero product property Square root property Complete the square Quadratic formula
2.6 Additional Equation-Solving Techniques	2.6.a. Students should be able to solve equations involving radicals. 2.6.b. Students should be able to solve equations involving quadratic-type polynomials.	Discriminant Demand Price-demand equation Revenue Extraneous solutions Quadratic type equations

Assessments

Homework – Students will be required to show work which reinforces classroom concepts. Homework will be evaluated for completeness (including level of documentation of work). It is used as a tool for multiple types of assessment. It will be used to formally assess if additional instruction is required and, at times, as a grade.

Class Notebook Checks – Students will maintain a formal set of student notes aligned to learning outcomes. They will be evaluated for completeness with level of documentation considered.

Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Suggested Strategies to Support Design of Coherent Instruction

Charlotte Danielson's Framework for Teaching: Domain 3 Instruction

3a – Student assignment sheets communicate expectations for learning.
3c – Instructional materials and unit project activities engage students in learning.
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Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook and online formats

Interdisciplinary Connections:

Economics
Physics
Chemistry
Geometry
Construction/Architecture
Business

Additional Resources:

Kahn Academy

Created By:
Rebecca Myers

Transitions to College Mathematics / 11-12 / Graphs

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Graphs	15 days

Grade Level Units
1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title
Graphs

Unit Overview
"Equations and inequalities are algebraic objects. A graph, on the other hand, is a geometric object such as a line, circle, or parabola. The idea of visualizing an equation or inequality by means of a graph was crucial to the development of analytic geometry, a subject that combines algebra nd geometry. In this chapter, we study the fundamentals of analytic geometry: The Cartesian coordinate system, name after the French mathematician and philosopher René Descartes (1596-1650); the calculation of distances in the plane; and equations of lines and circles. We conclude the chapter by applying linear models to solve real-world problems." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

3.1 How do you graph a point?
How do you use symmetry to aid in graphing?

3.2 How do you find the distance between two points?
How do you find the midpoint between two points?
How do you write the equation of a circle?

3.3 What is slope?
How do you write the equation of a line?
How do you write the equations for parallel and perpendicular lines?

3.4 How do you find a linear regression model?

Key Understandings

1. Graphing
2. Writing equations
3. Characteristics of slope
4. Finding regression models

Focus Standards Addressed in the Unit

CC.2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC.2.2.HS.C.6	Interpret functions in terms of the situations they model.

CC.2.4.HS.B.3	Analyze linear models to make interpretations based on the data.

Important Standards Addressed in the Unit

Misconceptions	**Proper Conceptions**
1. When testing for symmetry algebraically, The square of a negative number and the negative of a square number are the same: $(-x)^2 = -x^2$	1. The square of a negative number is a positive number and the negative of a square number is negative: $(-x)^2 = x^2$ AND $-(x^2) = -x^2$.
2. When finding the center of a circle given the standard form of the equation, *h* and *k* are the same signs given in the formula: $(x-3)^2 + (y+2)^2 = 4 \rightarrow C:(-3,2)$	2. When finding the center of a circle given the standard form of the equation, *h* and *k* are the opposite signs given in the formula: $(x-3)^2 + (y+2)^2 = 4 \rightarrow C:(3,-2)$
3. The *b* in the slope-intercept form of a linear equation is the same as the *B* in the standard form of the linear equation.	3. The *b* in the slope-intercept form of a linear equation is the *y*-intercept of the line whereas the *B* in the standard form of the linear equation is only a coefficient.

Concepts	**Competencies**	**Vocabulary**
3.1 Cartesian Coordinate Systems	3.1.a. Students should be able to identify the components and vocabulary associated with the Cartesian coordinate system. 3.1.b. Students should be able to graph equations using individual points. 3.1.c. Students should be able to use symmetry to aid in graphing.	Cartesian Coordiante system Rectangular Coordinate system Horizontal axis Vertical axis Quadrants
3.2 Distance in the Plane	3.2.a. Students should be able to use the distance formula to find the distance between two points 3.3.b. Students should be able to use the midpoint formula to find the midpoint between two points 3.3.c. Students should be able to write the standard form of the circle from multiple sources.	Abscissa Ordinate Origin Graph Solution Solution set Graph of an equation in two variables Point-by-point plotting Reflection Symmetry Distance Midpoint Circle Radius Center
3.3 Equations of Line	3.3.a. Students should be able to graph a line from multiple representations. 3.3.b. Students should be able to find the slope of a line given multiple representations. 3.3.c. Students should be able to convert a linear equation into multiple forms. 3.3.d. Students should be able to find the equation of parallel and perpendicular lines.	
3.4 Linear Equations and Models	3.4.a. Students should be able to interpret slope as a rate of change. 3.4.b. Students should be able to interpret linear models for real-world applications. 3.4. c. Students should be able to create linear regressions to model real-world problems.	Standard form of the circle equation Standard form of a linear equation y-intercept x-intercept rise run slope slope-intercept form point-slope form parallel perpendicular mathematical model linearly related

		rate of change speed velocity average rate of change regression analysis curve fitting curves scatter plot extrapolation interpolation predictions linear regression regression line

Assessments

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Quizzes – Within each unit, competencies will be assessed in smaller chunks as a grade and for the purpose of evaluating student understanding.

Unit Test – Each unit will include a summative written test.

Unit Project – This unit will include a project which combines the use of technology with concepts learned throughout the chapter.

Suggested Strategies to Support Design of Coherent Instruction

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Interdisciplinary Connections:

Economics
Physics
Chemistry
Geometry
Construction/Architecture
Business

Additional Resources:

Kahn Academy

Created By:

Rebecca Myers

Transitions to College Mathematics / 11-12 / Functions

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Functions	28 days

Grade Level Units
1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title
Functions

Unit Overview
"The function concept is one of the most important ideas in mathematics. To study math beyond the elementary level, you absolutely need to have a solid understanding of functions and their graphs. In this chapter, you'll learn the fundamentals of what functions are all about, and how to apply them." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

4.1 What is a function?

4.2 How do you graph linear functions?
How do you graph piecewise functions?

4.3 How do the properties of transformations connect equations and graphs?
What are even and odd functions?

4.4 How do you graph quadratic functions?
How do you solve quadratic inequalities?
How do you find a quadratic regression model?

4.5 How do you add, subtract, and multiply functions?
How do you perform a composition of functions?

4.5 What are one-to-one functions?
How do you find inverse functions?
How do you graph inverse functions?

Key Understandings

1. The distinction between a function and a relation
2. Function notation
3. Graphing: linear, quadratic, and inverse functions
4. Transformations of functions
5. Finding regression models
6. Compositions of functions
7. Inverse functions

Focus Standards Addressed in the Unit	
CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.7	Create and graph equations or inequalities to describe numbers or relationships.
CC.2.2.HS.C.4	Interpret the effects transformations have on functions and find the inverses of functions.
CC.2.2.HS.C.1	Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.1.HS.F.7	Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Misconceptions	Proper Conceptions
1. $f(x)$ means f times x.	1. $f(x)$ is the function notation that means the function f evaluated at each point x.
2. $f(x+c)$ translates the graph c units to the right and $f(x-c)$ translates the graph c units to the left.	2. $f(x+c)$ translates the graph c units to the left and $f(x-c)$ translates the graph c units to the right.
3. Domain describes the limits on the y-values of a function. (misconception is most commonly made when using graphical representations)	3. Domain describes the limits on the x-values of a function.
4. In order to solve quadratic inequalities, you must set each factor greater than/less than zero (which is the same process as solving quadratic equations).	4. In order to solve quadratic inequalities, you must use sign analysis on a number line or graph (which differs from the process for solving quadratic equations).
5. $f \circ g$ means f times g.	5. $f \circ g$ means to compose functions f and g: $f(g(x))$.
6. $f^{-1}(x)$ means $1/f(x)$.	6. $f^{-1}(x)$ is the notation for an inverse function.

Concepts	Competencies	Vocabulary
4.1 Functions	4.1.a. Students should be able to define and identify functions from multiple representations. 4.1.b. Students should be able to utilize function notation. 4.1.c. Students should be able to apply the concept of functions to real-world problems.	Functions Domain Range Independent variable Dependent variable Implied domain
4.2 Graphing Functions	4.2.a. Students should be able to graph linear functions given multiple representations 4.2.b. Students should be able to graph piecewise-defined functions.	f(x) difference quotient graphing a function zero root
4.3 Transformations of Functions	4.3.a . Students should be able to identify basic parent functions. 4.3.b. Students should be able to use transformations of functions	increasing decreasing

	to shift graphs horizontally and vertically. 4.3.c. Students should be able to use transformations of functions to reflect graphs. 4.3.d. Students should be able to use transformations of functions to stretch and shrink graphs. 4.3.e. Students should be able to identify even and odd functions and use the properties of even and odd functions to aid in graphing.	constant constant function identity function absolute-value function piece-wise function continuous greatest integer function transformation vertical shift horizontal shift reflection rigid transformation non-rigid transformation expansion contraction even function odd function symmetry vertex parabola general form of a quadratic equation standard form of a quadratic equation complete the square axis of symmetry maximum minimum quadratic inequality break-even points domain composition of functions one-to-one function inverse function
4.4. Quadratic Functions	4.4.a. Students should be able to graph quadratic functions. 4.4.b. Students should be able to write the equation of a parabola given multiple representations. 4.4.c. Students should be able to apply the concepts of quadratic equations to solve real-world problems. 4.4.d. Students should be able to solve quadratic inequalities. 4.4.e. Students should be able to quadratic regressions to model real-world problems.	
4.5 Operations on Functions; Compositions	4.5.a. Students should be able to operate (add, subtract, multiply, divide) on functions. 4.5.b. Students should be able to compose multiple functions. 4.5.c. Students should be able to apply the concepts of functions to solve real-world problems.	
4.6 Inverse Functions	4.6.a. Students should be able to identify one-to-one functions given multiple representations. 4.6.b. Students should be able to find inverse functions. 4.6.c. Students should be able to apply the concepts of inverse functions to solve real-world problems. 4.6.d. Students should be able to graph inverse functions.	

Assessments

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Unit Test – Each unit will include a summative written test.

Unit Project – This unit will include a project which combines the use of technology with concepts learned throughout the chapter.

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Interdisciplinary Connections:

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Geometry
Construction/Architecture
Business

Additional Resources:

Kahn Academy

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Transitions to College Mathematics / 11 - 12 / Polynomial and Rational Functions

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Polynomial and Rational Functions	20 days

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
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10 – Additional Topics in Analytic Geometry
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12 – Sequences, Induction, and Probability

Unit Title

Polynomial and Rational Functions

Unit Overview

"To model more complicated phenomena, [in this chapter] we will study the more general class of polynomial functions…A polynomial function can have many turning point. We will investigate the graphs and zeros of polynomial and apply that knowledge to study functions that can be written as quotients of polynomials, that is the rational functions. Finally, we will use the language of variation to describe a wide range of mathematical models used in engineering and the physical, social, and health sciences. (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

5.1 What are the properties of polynomial functions?
How do you graph polynomial functions?
How do you divide polynomials?
What are the remainder and factor theorems?

5.2 What are the methods for finding the real zeros of a polynomial?
How do you solve polynomial inequalities?

5.3 How do you find all zeros of a polynomial?

5.4 What are the properties of rational functions?
How do you find vertical, horizontal, and oblique asymptotes of a rational function?
How do you graph rational functions?
How do you solve rational inequalities?

5.5 What are direct, inverse, joint, and combined variation models?

Key Understandings

1. Graphing: polynomial and rational functions
2. Solving polynomial equations that are unfactorable
3. Solving inequalities: polynomial and rational functions
4. Using variation models to solve problems

Focus Standards Addressed in the Unit	
CC.2.2.HS.D.4	Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Misconceptions	Proper Conceptions
1. In polynomial division, when a term is missing in either the dividend or the divisor, no place holders need to be added before division: $(x^2+1)\div(x+1)$	1. In polynomial division, when a term is missing in the either the dividend or the divisor, a zero place holder needs to be added before division: $(x^2+0x+1)\div(x+1)$
2. In order to solve polynomial inequalities, you must set each factor greater than/less than zero (which is the same process as solving polynomial equations).	2. In order to solve polynomial inequalities, you must use sign analysis on a number line or graph (which differs from the process for solving polynomial equations).
3. Horizontal and oblique asymptotes can occur on the same graph.	3. Horizontal and oblique asymptotes cannot occur on the same graph.
4. Values for x that make the denominator equal to zero are in the domain of the function.	4. Values for x that make the denominator equal to zero are not in the domain of the function and create vertical asymptotes on the graph.

Concepts	Competencies	Vocabulary
4.1 Polynomial Functions, Division, and Models	5.1.a. Students should be able to identify the properties of polynomial functions given multiple representations. 5.1.b. Students should be able to divide polynomials using both long and synthetic division. 5.1.c. Students should be able to apply the remainder and factor theorems when using polynomial division. 5.1.d. Students should be able to apply the concepts of polynomials to solve real-world problems.	Zero Root Degree of a polynomial Coefficient Turning point Leading term End behavior Polynomial long division Synthetic division
4.2 Real Zeros and Polynomial Inequalities	5.2.a. Students should be able to find upper and lower bounds on real zeros of a polynomial. 5.2.b. Students should be able to use the location theorem and bisection methods to find real zeros of a polynomial function. 5.2.c. Students should be able to approximate real zeros and turning points of polynomial functions. 5.2.d. Students should be able to solve polynomial inequalities. 5.2.e. Students should be able to apply the concepts of polynomial zeros and turning points to solve real-world problems.	Quotient Divisor Remainder Remainder theorem Factor theorem Upper bound Lower bound Location theorem Bisection method Multiplicity
4.3 Complex Zeros and Rational Zeros of Polynomials	5.3.a. Students should be able to apply the fundamental theorem of algebra to polynomial functions. 5.3.b. Students should be able to factor polynomials with both real and complex coefficients. 5.3.c. Students should be able to graph polynomials. 5.3.d. Students should be able to find rational zeros of polynomial functions.	Double root Triple root Rational zero theorem Rational function Vertical asymptote Horizontal asymptote Oblique asymptote Slant asymptote
4.4 Rational Functions and Inequalities	5.4.a. Students should be able to identify the properties of rational functions. 5.4.b. Students should be able to find vertical and horizontal	Direct variation Inverse variation Joint variation

	asymptotes given multiple representations. 5.4.c. Students should be able to analyze the graphs of rational functions given multiple representations. 5.4.d. Students should be able to solve rational inequalities.	Combined variation Constant of proportionality
4.5 Variation and Modeling	5.5.a. Students should be able to identify and write direct variation models 5.5.b. Students should be able to identify and write inverse variation models. 5.5.c. Students should be able to identify and write joint and combined variation models. 5.5.d. Students should be able to apply the concepts of variation modeling to solve real-world problems.	

Assessments

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Unit Test – Each unit will include a summative written test.

Suggested Strategies to Support Design of Coherent Instruction

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Differentiation:

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Interdisciplinary Connections:

Economics
Physics
Chemistry
Geometry
Construction/Architecture
Business

Additional Resources:

Kahn Academy

Created By:

Rebecca Myers

Transitions to College Mathematics / 11 - 12 / Exponential and Logarithmic Functions

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Exponential and Logarithmic Functions	25 days

Grade Level Summary

This course is designed to bridge the gap between Algebra II, Geometry, and collegiate courses in mathematics. Emphasis will be placed on linear, polynomial, rational, trigonometric, exponential, and logarithmic functions, matrices, systems of equations and inequalities, and other algebraic and geometric concepts. Students may use graphing calculators and computer software for various mathematical applications. Both topics and depth of study aim to be consistent with the expectations of a traditional College Algebra course required for many non-math dependent collegiate majors or many associate degree programs.

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Exponential and Logarithmic Functions

Unit Overview

In [this chapter] we will study exponential and logarithmic functions. These functions are not algebraic; they belong to the class of transcendental functions. Exponential and logarithmic functions are use to model a surprisingly wide variety of real-world phenomena; growth of populations of people, animals, and bacteria; decay of radioactive substances; epidemics; magnitudes of sounds and earthquakes. These and many other applications will be studied in this chapter. (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

6.1 What are the properties of exponential functions?
How do you graph exponential functions?
What is the number *e*?
How do you solve problems involving compound interest?

6.2 How do you use exponential functions to solve real world problems?
How do you find exponential regression models?

6.3 What are the properties of logarithmic functions?
What are common and natural logarithms?

6.4 How do you use logarithmic equations to solve real world problems?
How do you find logarithmic regressions?

Key Understandings

1. Relationships between exponential and logarithmic equations
2. Solving exponential and logarithmic equations
3. Graphing: exponential equations

6.5 How do you solve exponential equations? How do you solve logarithmic equations?	

Focus Standards Addressed in the Unit

CC.2.2.HS.C.5	Construct and compare linear, quadratic, and exponential models to solve problems.

Important Standards Addressed in the Unit

Misconceptions	**Proper Conceptions**
1. $\log M + \log N = \log(M + N)$	1. $\log M + \log N = \log(MN)$
2. $\log M - \log N = \dfrac{\log M}{\log N}$	2. $\log M - \log N = \log \dfrac{M}{N}$
3. $(\log x)^2 = \log x^2$	3. $(\log x)^2 = (\log x)(\log x) = 2(\log x)$

Concepts	**Competencies**	**Vocabulary**
6.1 Exponential Functions	6.1.a. Students should be able to identify the properties of exponential functions. 6.1.b. Students should be able to graph exponential functions given multiple representations. 6.1.c. Students should be able to use exponential functions to calculate compound interest	Exponential function Base Base *e* Interest rate Interest Principal Compound interest
6.2 Exponential Models	6.2.a. Students should be able to use exponential functions to solve real-world problems. 6.2.b. Students should be able to use exponential regressions to model real-world problems. 6.2.c. Students should be able to compare exponential models in terms of limited and unlimited growth.	Continuously compounded interest formula Doubling time Growth model Relative growth rate Half-life Negative growth
6.3 Logarithmic Functions	6.3.a. Students should be able to identify the properties of logarithmic functions. 6.3.b. Students should be able to convert between exponential and logarithmic forms. 6.3.c. Students should be able to use common and natural logarithms. 6.3.d. Students should be able to use the change-of-base formula.	Decay model Learning curve Limited growth Unlimited growth Logistic growth Logarithmic function Common logarithm Natural logarithm
6.4 Logarithmic Models	6.4 a. Students should be able to apply the concepts of logarithms to solve real-world problems. 6.4.b. Students should be able to use logarithmic regressions to model real-world problems.	Change of base formula Decibel level Magnitude Richter scale catenary
6.5 Exponential and Logarithmic Equations	6.5.a. Students should be able to solve exponential equations. 6.5.b. Students should be able to solve logarithmic equations.	

Assessments

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Interdisciplinary Connections:

Economics
Physics
Chemistry
Geometry
Construction/Architecture
Business

Additional Resources:

Kahn Academy

Created By:

Rebecca Myers

Transitions to College Mathematics / 11 - 12 / Trigonometric Functions

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Trigonometric Functions	28 days

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Trigonometric Functions

Unit Overview

"Originally the trigonometric functions were restricted to angels and their applications to the indirect measurement of angles and distances. These functions gradually broke free of these restrictions, and we now have trigonometric functions of real numbers. Modern application range over many types of problems that have little or nothing to do with angles or triangles—applications involving periodic phenomena such as a sound, light, and electrical waves; business cycles; and planetary motion. In our approach to the subject, we define the trigonometric function s both in terms of angles, and coordinates of point on the unit circle. (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions	Key Understandings
7.1 How do you convert between radians and degrees? 7.2 What are the six trigonometric ratios? How do you solve a right triangle? 7.3 How are the trigonometric functions defined? How do you graph the trigonometric functions? 7.4 What the basic trigonometric identities? How do you find a reference angle? 7.5 How do you use transformations of functions to graph the trigonometric functions? How do you find the equation from the graph of a trigonometric function? 7.6 What are the inverse trigonometric functions?	1. The distinction between radians and degrees 2. Solving right triangles 3. Trigonometric functions and their usage 4. Graphing: trigonometric functions

Focus Standards Addressed in the Unit

CC.2.2.HS.D.1	Interpret the structure of expressions to represent a quantity in terms of its context.

CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.C.7	Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.
CC.2.2.HS.C.8	Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.
CC.2.3.HS.A.7	Apply trigonometric ratios to solve problems involving right triangles.

Important Standards Addressed in the Unit	
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.

Misconceptions	**Proper Conceptions**
1. $(\sin x)^2 = \sin x^2$	1. $(\sin x)^2 = \sin^2 x$
2. $(\cos x)^2 = \cos x$	2. $(\cos x)^2 = \cos^2 x$

Concepts	**Competencies**	**Vocabulary**
7.1 Angles and Their Measures	7.1.a. Students should be able to classify angles. 7.1.b. Students should be able to convert angles measures between radians and degrees. 7.1.c. Students should be able to apply the concepts of angles to calculate linear and angular speed.	Angle Initial angle Terminal side Vertex Negative angle Positive angle
7.2 Right Triangle Trigonometry	7.2.a. Students should be able to identify the trigonometric ratios. 7.2.b. Students should be able to evaluate the trigonometric ratios. 7.2.c. Students should be able to solve right triangles.	Coterminal Standard position Quadrantal angle Degree
7.3 Trigonometric Functions; A unit Circle Approach	7.3.a. Students should be able to use the unit circle to identify circular coordinates. 7.3.b. Students should be able to define trigonometric functions using the unit circle. 7.3.c. Students should be able to graph trigonometric functions.	Straight angle Right angle Obtuse angle Acute angle Complementary Supplementary
7.4 Properties of Trigonometric Functions	7.4.a. Students should be able to simplify trigonometric functions using the basic trigonometric identities. 7.4.b. Students should be able to identify the sign properties of the basic trigonometric functions given the quadrant. 7.4.c. Students should be able to identify the periods of the basic trigonometric functions.	Seconds Minutes Radian Linear speed Angular speed Right triangle Trigonometric ratios
7.5 More General Trigonometric Functions and Models	7.5.a. Students should be able to graph sine and cosine functions given an amplitude and period. 7.5.b. Students should be able to graph sine and cosine functions given amplitude, period, and horizontal shifts. 7.5.c. Students should be able to find the equation of a simple harmonic. 7.5.d. Students should be able to use sinusoidal regressions to model real-world problems.	Sine Cosine tangent Cotangent Secant Cosecant Reciprocal relationships Complementary relationships
7.6 Inverse Trigonometric Functions	7.6.a. Students should be able to derive inverse sine, inverse cosine, and inverse tangent functions. 7.6.b. Students should be able to graph inverse sine, inverse cosine, and inverse tangent functions.	Cofunction Side opposite Side adjacent Hypotenuse Solve (a right triangle)

		Wrapping function Circular point Reference triangle Reference angle Periodic Fundamental period of *f* Simple harmonic motion Simple harmonics Sinusoidal regression Inverse sine Inverse cosine Inverse tangent

Assessments

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Interdisciplinary Connections:

Economics
Physics
Chemistry
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Business

Additional Resources:

Kahn Academy

Created By:

Rebecca Myers

Transitions to College Mathematics / 11 - 12 / Trigonometric Identities and Conditional Equations

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Trigonometric Identities and Conditional Equations	25 days

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Trigonometric Identities and Conditional Equations

Unit Overview

"Trigonometric functions are widely used in solving real-world problems and in the development of mathematics. Whatever their use, it is often of value to be able to change a trigonometric expression form one form to an equivalent form. This involves the use of identities…[in this chapter we] deal with trigonometric identities, and…conditional trigonometric equations." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions	Key Understandings
8.1 What are the basic trigonometric identities? 8.2 What are the sum, difference, and cofunction identities? 8.3 What are the half-angle and double-angle identities? 8.4 What are the product-sum and sum-product identities? 8.5 How do you solve trigonometric equations algebraically? How do you solve trigonometric equations graphically?	1. Identifying and using trigonometric identities to solve equations.

Focus Standards Addressed in the Unit

CC.2.2.HS.C.8	Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.
CC.2.2.HS.C.9	Prove the Pythagorean identity and use it to calculate trigonometric ratios.

Important Standards Addressed in the Unit

Misconceptions	Proper Conceptions

Concepts	Competencies	Vocabulary
8.1 Basic Identities and Their Use	8.1.a. Students should be able to algebraically prove the trigonometric identities.	Difference identify for cosine Sum identity for cosine
8.2 sum, Difference, and Cofunction identities	8.2.a. Students should be able to apply the difference and sum identities for sine, cosine, and tangent. 8.2.b. Students should be able to apply the Cofunction identities for sine, cosine, and tangent.	Cofunction identities for sine Cofunction identity for tangent Cofunction identity for cosine
8.3 Double-Angle and Half-angle Identities	8.3.a. Students should be able to apply the double-angle identities for sine and cosine. 8.3.b. Students should be able to apply the half-angle formulas for sine and cosine.	Product-sum identities Sum-product identities Identity equation Conditional equation
8.4 Product-Sum and Sum-Production Identities	8.4.a. Students should be able to apply the product-sum identities. 8.4.b. Students should be able to apply the sum-product identities.	
8.5 Trigonometric Equations	8.5.a. Students should be able to solve trigonometric equations algebraically. 8.5.b. Students should be able to solve trigonometric equations graphically.	

Assessments

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Chemistry
Geometry
Construction/Architecture

Business

Additional Resources:
Kahn Academy

Created By:
Rebecca Myers

Transitions to College Mathematics / 11 - 12 / Additional Topics in Trigonometry

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Additional Topics in Trigonometry	25 days

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Additional Topics in Trigonometry

Unit Overview

"In [this chapter] a number of additional topics involving trigonometry are considered. First, we return to the problem of solving triangles, not just right triangles, but any triangle. The some of these ideas are used to develop the important concept of vector. With our knowledge of trigonometry, we introduce the polar coordinate system probably the most important coordinate system after the rectangular coordinate system. After considering polar equations and their graphs, we represent complex numbers in polar form. Once a complex number is in polar form, it will be possible to find *n*th powers and *n*th roots of the number using an ingenious theorem established by De Moivre." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

9.1 When can you use the Law of Sines to solve triangles?

9.2 When can you use the Law of Cosines to solve triangles?

9.3 What is a vector?
What are the properties of vectors?
How do you add and multiply vectors?

9.4 How do you convert between polar and rectangular coordinates?
How do you graph polar coordinates?
How can you use polar coordinates to solve real-world problems?

9.5 How do you plot points in the complex plane?
How do you multiply and divide in the polar form?
What is DeMoivre's Theorem?

Key Understandings

1. Using Laws of Sines and Cosines to solve triangles.
2. Vector addition and multiplication
3. Using the polar coordinate system
4.

Focus Standards Addressed in the Unit	
CC.2.1.HS.F.6	Extend the knowledge of arithmetic operations and apply to complex numbers.
CC.2.1.HS.F.7	Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.

Important Standards Addressed in the Unit	

Misconceptions	Proper Conceptions

Concepts	Competencies	Vocabulary
9.1 Law of Sines	9.1.a. Students should be able to derive the law of sines. 9.1.b. Students should be able to solve AAS and ASA triangles 9.1.c. Students should be able to solve SSA (ambiguous case) triangles.	Law of sines Oblique triangle Acute Obtuse Law of cosines
9.2 Law of Cosines	9.2.a. Students should be able to derive the law of cosines. 9.2.b. Students should be able to solve SAS and SSS triangles.	Vector Magnitude Same direction
9.3 Students should be able to ectors in the Plane	9.3.a. Students should be able to identify the components of a vector. 9.3.b. Students should be able to add and multiply vectors. 9.3.c. Students should be able to find unit vectors. 9.3.d. Students should be able to apply the concepts of vectors to velocity. 9.3.e. Students should be able to apply the concepts of vectors to force.	Opposite direction Zero vector Equal vectors Standard vectors Scalar components Normal vector Tail-to-tip rule Parallelogram rule Resultant
9.4 Polar coordinates and Graphs	9.4.a Students should be able to identify the components of the polar coordinate system. 9.4.b. Students should be able to convert between polar and rectangular coordinates 9.4.c. Students should be able to graph polar equations. 9.4.d. Students should be able to identify basic polar curves 9.4.e. Students should be able to apply the concepts of polar curves to solve real-world problems.	Vector components Scalar product Force vector Resultatnt force Apparent velocity Actual velocity Static equilibrium Polar coordinate system Pole
9.5 Complex Numbers and De Moivre's Theorem	9.5.a. Students should be able to plot numbers on the complex plane. 9.5.b. Students should be able to covert between polar and complex rectangular coordinates. 9.5.c. Students should be able to multiply and divide complex numbers in polar form 9.5.d. Students should be able to apply De Moivre's theorem. 9.5.e. Students should be able to find distinct roots of complex numbers.	Origin Polar axis Graph Point-by-point plotting Rapid polar sketching Cardioid Four-leafed rose Complex axis Real axis Imaginary axis Rectangular form Polar form Modulus Absolute value Mod

		Argument *n*th root of *z*

Assessments

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Additional Resources:

Kahn Academy

Created By:

Rebecca Myers

Transitions to College Mathematics / 11 - 12 / Additional Topics in Analytic Geometry

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Additional Topics in Analytic Geometry	25 days

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Additional Topics in Analytic Geometry

Unit Overview

"Analytic geometry is the study of geometric objects using algebraic techniques. René Descartes (1596-1650), the French philosopher and mathematician, is generally recognized as the found of the subject. We used analytic geometry…to obtain equations of lines and circles. In [this chapter], we take a similar approach to the study of parabolas, ellipses, and hyperbolas. Each of these geometric objects is a conic section, that is, the intersection f a plane and a cone. We will derive equations for the conic sections and explore a wealth of application in architecture, communications, engineering, medicine, optics, and space science." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

10.1 What is a conic section?
How do you write the equation of a parabola as defined by a conic section?

10.2 What is an ellipse?
How do you write the equation of an ellipse as defined by a conic section?

10.3 What is a Hyperbola?
How do you write the equation of a hyperbola as defined by a conic section?

10.4 How are translations used in graphing?
How are rotations used in graphing?

Key Understandings

1. Graphing: parabolas, ellipses, and hyperbolas

Focus Standards Addressed in the Unit

CC.2.3.HS.A.14	Apply geometric concepts to model and solve real world problems.

CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.
CC.2.2.HS.D.5	Use polynomial identities to solve problems.
CC.2.2.HS.C.2	Graph and analyze functions and use their properties to make connections between the different representations.
CC.2.3.HS.A.10	Translate between the geometric description and the equation for a conic section.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
CC.2.2.HS.C.6	Interpret functions in terms of the situations they model.

Misconceptions	**Proper Conceptions**

Concepts	**Competencies**	**Vocabulary**
10.1 Conic Sections: Parabola	10.1.a. Students should be able to identify conic sections 10.1.b. Students should be able to graph parabolas. 10.1.c. Students should be able to apply the concepts of parabolas to solve real-world problems.	Parabola Hyperbola Degenerate conic Coordinate-free definition Focus Directrix Axis of symmetry Vertex Ellipse Circle Nappes Right circular cone Paraboloid Foci Minor axis Major axis Hyperbola Transvers axis Center Conjugate axis Asymptote rectangle Translation of coordinate axes Rotation of coordinate axes discriminant
10.2 Ellipse	10.2.a. Students should be able to graph ellipses. 10.2.b. Students should be able to apply the concepts of ellipses to solve real-world problems.	
10.3 Hyperbola	10.3.a. Students should be able to graph hyperbolas. 10.3.b. Students should be able to apply the concepts of hyperbolas to solve real-world problems.	
10.4 Translation and Rotation of Axes	10.4.a. Students should be able to identify translations of a curve. 10.4.b. Students should be able to use translations of a curve to aid in graphing. 10.4.c. Students should be able to identify rotations of a curve. 10.4.d. Students should be able to use rotations of a curve to aid in graphing. 10.4.e. Students should be able to identify conic sections based on their equations.	

Assessments

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Suggested Strategies to Support Design of Coherent Instruction

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Differentiation:

- Provide graphic organizers
- Provide multiple concrete examples
- Permit projects to be complete over extended time period
- Provide lesson notes via visual (smart board) as well as in notebook and online formats

Interdisciplinary Connections:

Economics
Physics
Chemistry
Geometry
Construction/Architecture
Business

Additional Resources:

Kahn Academy

Created By:

Rebecca Myers

Transitions to College Mathematics / 11 - 12 / Systems of Equations and Matrices

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Systems of Equations and Matrices	25 days

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
3 – Graphs
4 - Functions
5 – Polynomial and Rational Functions
6 – Exponential and Logarithmic Functions
7 – Trigonometric Functions
8 – Trigonometric Identities and Conditional Equations
9 – Additional Topics in Trigonometry
10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Systems of Equations and Matrices

Unit Overview

"We have seen many real-world situations where solving an equation is valuable. But the world is a very complicated place, and may more situations lead to more than one variable. In that case, solving a system of equations becomes important. In this chapter, we will study a variety of methods for solving systems of equation. We will begin with linear systems with two or three variables using algebraic techniques similar to those we used for solving individual equations. Then we will introduce a variety of matrix methods for solving linear systems. These methods can be applied to very large systems that model very complicated real-world problems." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

11.1 How do you solve a system of equations graphically?
How do you solve a system of equations algebraically?

11.2 How do you solve a system of equations using Gauss-Jordan elimination?

11.3 How do you add, subtract, and multiply matrices?

11.4 How do you find the identity of a matrix?
How do you find the inverse of a matrix?
How do you solve matrix equations?

11.5 How do you find first- and second-order determinants?
How do you use solve a system of equations using Cramer's Rule?

Key Understandings

1. Solving systems of equations
2. Properties of matrices
3. Operating with matrices

Focus Standards Addressed in the Unit

CC.2.2.HS.D.10	Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

CC.2.2.HS.D.8	Apply inverse operations to solve equations or formulas for a given variable.

Important Standards Addressed in the Unit	

Misconceptions	**Proper Conceptions**

Concepts	**Competencies**	**Vocabulary**
11.1 Systems of Linear Equations	11.1.a. Students should be able to solve systems of linear equations by graphing. 11.1.b. Students should be able to solve systems of linear equations by substitution. 11.1.c. Students should be able to solve systems of linear equations by elimination. 11.1.d. Students should be able to apply the concepts of systems of linear equations to solve real-world problems.	System of linear equations in two variables Coefficients Constant terms Solution Solution set Consistent Inconsistent Independent
11.2 Solving Systems of Linear Equations Using Gauss-Jordan Elimination	11.2.a. Students should be able to reduce matrices. 11.2.b. Students should be able to solve systems by Gauss-Jordan elimination. 11.2.c. Students should be able to use matrices to solve real-world problems.	Unique solution Dependent Substitutions method Elimination by adding Equivalent systems Parameter
11.3 Matrix Operations	11.3.a. Students should be able to add, subtract, and multiply matrices 11.3.b. Students should be able to multiply a matrix by a constant.	Particular solution Equilibrium price Equilibrium quantity Matrix
11.4 Solving Systems of Linear Equations Using Matrix Inverse Methods	11.4.a. Students should be able to find the identity of a matrix. 11.4.b. Students should be able to find the inverse of a square matrix. 11.4.c. Students should be able to identify and use the properties of matrices. 11.4.d. Students should be able to use matrix equations to solve systems of equations.	Element Size of a matrix Dimensions of a matrix Square matrix Column matrix Row matrix Double subscript notation Principal diagonal of a matrix
11. 5 Determinants and Cramer's Rule	11.5.a. Students should be able to define first and second order determinants. 11.5.b. Students should be able to evaluate third order determinants. 11.5.c. Students should be able to use Cramer's Rule to solve systems of equations.	Augmented coefficient matrix Coefficient matrix Constant matrix Row-equivalent Row operations Reduce row echelon form Reduced system Submatrix Gauss-Jordan Elimination Zero matrix Negative of a matrix Matrix product Identity for matrix multiplication Multiplicative inverse of a matrix Singular matrix Determinant First-order determinant

		Second-order determinant Secondary diagonal Third-order determinant Minor of an element Cofactor of an element Cramer's Rule

Assessments

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Additional Resources:

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Transitions to College Mathematics / 11 - 12 / Sequences, Inductions, and Probability

Subject	Grade	Unit	Suggested Timeline
Mathematics	11 - 12	Sequences, Inductions, and Probability	25 days

Grade Level Summary

This course is designed to bridge the gap between Algebra II, Geometry, and collegiate courses in mathematics. Emphasis will be placed on linear, polynomial, rational, trigonometric, exponential, and logarithmic functions, matrices, systems of equations and inequalities, and other algebraic and geometric concepts. Students may use graphing calculators and computer software for various mathematical applications. Both topics and depth of study aim to be consistent with the expectations of a traditional College Algebra course required for many non-math dependent collegiate majors or many associate degree programs.

Grade Level Units

1 – Basic Algebraic Operations
2 – Equations and Inequalities
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10 – Additional Topics in Analytic Geometry
11 – Systems of Equations and Matrices
12 – Sequences, Induction, and Probability

Unit Title

Sequences, Inductions, and Probability

Unit Overview

"Sequences, and the related concept of series, are useful tools in almost all areas of mathematics. In this chapter, they will play roles in the development of several topics: a method of proof called mathematical inductions, techniques for counting, and probability." (Barnett, Zielger, Byleen, Sobecki, *College Algebra with Trigonometry*, McGraw Hill, 2011.)

Unit Essential Questions

12.1 What are sequences and series?

12.2 How can you use mathematical induction to prove a conjecture?

12.3 What are arithmetic and geometric sequences?

12.4 What are combinations and permutations?

12.5 How do you find the probability of an event?

12.6 What are the applications for Pascal's Triangle? What is the Binomial formula?

Key Understandings

1. The distinctions between sequences and series
2. Using mathematical induction
3. The distinctions between arithmetic and geometric series
4. Finding probability.

Focus Standards Addressed in the Unit

CC.2.4.HS.B.4	Recognize and evaluate random processes underlying statistical experiments.

CC.2.4.HS.B.6	Use the concepts of independence and conditional probability to interpret data.
CC.2.4.HS.B.7	Apply the rules of probability to compute probabilities of compound events in a uniform probability model.

Important Standards Addressed in the Unit	
CC.2.2.HS.D.2	Write expressions in equivalent forms to solve problems.
CC.2.4.HS.B.5	Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.

Misconceptions	**Proper Conceptions**

Concepts	**Competencies**	**Vocabulary**
12.1 Sequences and Series	12.1.a. Students should be able to define a sequence. 12.1.b. Students should be able to define a series.	Sequence Terms of a sequence General term of a sequence
12.2 Mathematical Inductions	12.2.a. Students should be able to use counterexamples to prove conjectures to be false. 12.2.b. Students should be able to prove conjectures with mathematical induction.	Recursion formula Fibonacci Sequence Finite sequence Infinite sequence Series
12.3 Arithmetic and Geometric Sequences	12.3.a. Students should be able to identify arithmetic and geometric sequences. 12.3.b. Students should be able to develop nth term formulas. 12.3.c. Students should be able to develop sum formulas for finite arithmetic series. 12.3.d. Students should be able to develop sum formulas for finite geometric series. 12.3.e. Students should be able to develop sum formulas for infinite geometric series.	Infinite series Finite series Summation notations Summing index Alternating series Counterexample Conjecture Mathematical induction Lagrange's Four Square Theorem
12.4 Multiplication Principle, Permutations, and Combinations	12.4.a. Students should be able to count with the multiplication principle. 12.4.b. Students should be able to use factorial notation. 12.4.c. Students should be able to identify and evaluate permutations. 12.4.d. Students should be able to identify and evaluate combinations.	Fermat's Last theorem Goldbach's Conjecture Arithmetic Sequence Geometric Sequence Common difference Common ratio Sum of an infinite geometric series
12.5 Sample Spaces and Probability	12.5.a. Students should be able to find the probability of an event. 12.5.b. Students should be able to make equally likely assumptions. 12.5.c. Students should be able to approximate empirical probability.	Multiplier doctrine Counting technique Tree diagram *n* factorial Zero factorial Permutation
12.6 The Binomial Formula	12.6.a. Students should be able to use Pascal's Triangle. 12.6.b. Students should be able to use the Binomial Formula.	Ordering Combination Random experiments Sample space Event Simple event Compound event Fundamental sample space Probability of an event Acceptable probability

		assignment Probability function Theoretical probability Empirical probability Actual probability Frequency Relative frequency Expected frequency Pascal's Triangle Binomial coefficients Binomial Formula

Assessments

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